

thm_2Eiterate_2ESUM__LMUL (TMK4hCwXs6JdaP2B2jACjYfswD8aEftBwMk)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2.V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod \ A0 \ A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow c_2Epair_2EABS_prod \ A_27a \ A_27b \in ((ty_2Epair_2Eprod \ A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 7 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (\text{ap } (\text{c_2Ebool_2E_2F_5C } (2^{A_27a})))$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow c_2Epred_set_2EGSPEC \ A_27a \ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod \ A_27a \ 2)^{A_27b})}) \tag{3}$$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (\text{ap } P \ x)) \text{ then } (the \ (\lambda x. x \in A \wedge p \ x))$ of type $\iota \Rightarrow \iota$.

Definition 21 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Erealax_Ereal)$

Definition 22 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Definition 23 We define $c_Eiterate_ESum$ to be $\lambda A_27a : \iota.(ap\ (c_Eiterate_Eiterate\ A_27a\ ty_Erealax_Ereal))$

Definition 24 We define $c_Emarker_EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_EEnum_EZERO_REP : \iota$ be given. Assume the following.

$$c_EEnum_EZERO_REP \in \omega \tag{10}$$

Let $ty_EEnum_EEnum : \iota$ be given. Assume the following.

$$nonempty\ ty_EEnum_EEnum \tag{11}$$

Let $c_EEnum_EABS_num : \iota$ be given. Assume the following.

$$c_EEnum_EABS_num \in (ty_EEnum_EEnum^{\omega}) \tag{12}$$

Definition 25 We define c_EEnum_E0 to be $(ap\ c_EEnum_EABS_num\ c_EEnum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_EEnum_EEnum}) \tag{13}$$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal)}) \tag{14}$$

Definition 26 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t))))))
\end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))))
\end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0op \in ((A_27a^{A_27a})^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). \\
& (\forall V2s \in (2^{A_27b}). ((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\
& A_27a)\ V0op)\ V2s)\ V1f) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ (ap\ (\\
& c_2Epred_set_2EFINITE\ A_27b)\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport \\
& A_27b\ A_27a)\ V0op)\ V1f)\ V2s)))\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate \\
& A_27b\ A_27a)\ V0op)\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport\ A_27b\ A_27a) \\
& V0op)\ V1f)\ V2s))\ V1f)))\ (ap\ (c_2Eiterate_2Eneutral\ A_27a)\ V0op))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c_2Eiterate_2Eneutral\ ty_2Erealax_2Ereal)\ c_2Erealax_2Ereal_add) = \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). ((ap\ (ap\ (c_2Eiterate_2ESum \\
& A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a))\ V0f) = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \wedge (\forall V1x \in A_27b. (\forall V2f \in (ty_2Erealax_2Ereal^{A_27b}). \\
& (\forall V3s \in (2^{A_27b}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b) \\
& V3s)) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& A_27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal) \\
& (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V1x)\ V3s))\ (ap\ (ap\ (c_2Eiterate_2ESum \\
& A_27b)\ V3s)\ V2f))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ V2f\ V1x))\ (\\
& ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ V3s)\ V2f)))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((ap\ (ap \\
& (c_2Eiterate_2ESum\ A_27a)\ V0s)\ (\lambda V1n \in A_27a. (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\\
& \quad (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& \quad (\forall V2e \in A_27a.((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a\ V2e)\ V1s))) \Rightarrow \\
& \quad (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a\ V2e)\ V1s)))))) \Rightarrow \\
& \quad (\forall V3s \in (2^{A_27a}).((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a\ \\
& \quad \quad V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x) (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)) \Leftrightarrow ((V0x = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \vee \\
& \quad \quad (V1y = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x) (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_add \\
& \quad \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y)) (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad \quad V0x)\ V2z))))))
\end{aligned} \tag{38}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). \\
& \quad (\forall V1c \in ty_2Erealax_2Ereal.(\forall V2s \in (2^{A_27a}).((\\
& \quad ap\ (ap\ (c_2Eiterate_2ESum\ A_27a)\ V2s) (\lambda V3x \in A_27a.(ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad V1c) (ap\ V0f\ V3x)))) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1c) (ap\ (ap \\
& \quad \quad \quad (c_2Eiterate_2ESum\ A_27a)\ V2s)\ V0f))))))
\end{aligned}$$