

# thm\_2Eiterate\_2ESUM\_LT (TMc- che97XmwmwPDcCuUZ5F4YTzstYFHBByKh)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \ P))$

**Definition 4** We define `c_2Ebool_2E_T` to be  $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})$

**Definition 6** We define `c_2Ebool_2E_F` to be  $(\text{ap (c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 9** We define `c_2Ebool_2E_COND` to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (\text{ap (c_2Emin_2E_3D } (2^{A-27a})$

Let `c_2Enum_2E_ZERO_REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2E_ZERO\_REP \in \text{omega} \tag{1}$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \tag{2}$$

Let `c_2Enum_2E_ABS_num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2E\_ABS\_num \in (ty\_2Enum\_2Enum^{\text{omega}}) \tag{3}$$

**Definition 10** We define `c_2Enum_2E_0` to be  $(\text{ap } c\_2Enum\_2E\_ABS\_num \ c\_2Enum\_2E\_ZERO\_REP)$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (5)$$

**Definition 11** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (8)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E40\ t))$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (9)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (11)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax$

**Definition 15** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota. \lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}). (ap\ (c\_2Emin$

**Definition 16** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

**Definition 17** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b}})^{A\_27a}) \end{aligned} \quad (12)$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (13)$$

**Definition 19** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27a}).\lambda V$

**Definition 20** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 21** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 22** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ 2)$

**Definition 23** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 24** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27a}).\lambda V$

**Definition 25** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ ty\_2Erealax$

**Definition 26** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2E$

**Definition 27** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (ap$

Let  $c\_2Erealax\_2Etreax\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\
& V0t1) V1t2) = V1t2)))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). ((ap\ (ap\ (c\_2Eiterate\_2ESum \\
& A\_27a)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a))\ V0f) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \wedge (\forall V1x \in A\_27b. (\forall V2f \in (ty\_2Erealax\_2Ereal^{A\_27b}). \\
& (\forall V3s \in (2^{A\_27b}). ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27b) \\
& V3s)) \Rightarrow ((ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27b)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\
& A\_27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal) \\
& (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V1x)\ V3s))\ (ap\ (ap\ (c\_2Eiterate\_2ESum \\
& A\_27b)\ V3s)\ V2f))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ (ap\ V2f\ V1x))\ ( \\
& ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27b)\ V3s)\ V2f)))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\
& (\forall V1g \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V2s \in (2^{A\_27a}). \\
& (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V2s)) \wedge (\forall V3x \in A\_27a. \\
& ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V2s)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ V0f\ V3x))\ (ap\ V1g\ V3x)))))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ V0f))\ (ap\ (ap\ (c\_2Eiterate\_2ESum \\
& A\_27a)\ V2s)\ V1g)))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1x \in \\
& A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x) \\
& (ap\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\
& (c\_2Ebool\_2EIN\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\
& (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V1s)) \Rightarrow ((ap\ (ap \\
& (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EDELETE \\
& A\_27a)\ V1s)\ V0x)) = V1s))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\ & (2^{A\_27a}). ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EDELETE \\ & A\_27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0w \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\ & (\forall V2y \in ty\_2Erealax\_2Ereal. (\forall V3z \in ty\_2Erealax\_2Ereal. \\ & (((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\ & V2y)\ V3z)))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0w)\ V2y))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V1x)\ V3z)))))) \end{aligned} \quad (32)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\ & (\forall V1g \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V2s \in (2^{A\_27a}). \\ & (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V2s)) \wedge ((\forall V3x \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V2s)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\ & (ap\ V0f\ V3x))\ (ap\ V1g\ V3x)))))) \wedge (\exists V4x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V4x)\ V2s)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ V0f\ V4x)) \\ & (ap\ V1g\ V4x)))))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ (ap\ (c\_2Eiterate\_2ESum \\ & A\_27a)\ V2s)\ V0f))\ (ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ V1g)))))) \end{aligned}$$