

thm_2Eiterate_2ESUM__MULTICOUNT__GEN
(TMFdPF7bqnmcgKksieVET85zaxQETCS4pVU)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge P\ x)$ of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$.(ap $(c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0a))$).

Let $c_2Erealax_2Etreax_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealax_2Etreax_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$.

Definition 13 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 14 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota$. $\lambda V0op \in ((A_27a^{A_27a})^{A_27a})$.(ap $(c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0op))$).

Definition 15 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota$. $\lambda A_27b : \iota$. $\lambda V0op \in ((A_27b^{A_27b})^{A_27b})$. $\lambda V1op \in ((A_27a^{A_27a})^{A_27a})$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota$.($\lambda V0t \in 2$.($\lambda V1t1 \in A_27a$.($\lambda V2t2 \in A_27a$.(

Definition 17 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2$.($\lambda V1t2 \in 2$.(ap $(c_2Ebool_2E21\ 2)$ $(\lambda V2t \in 2$.(

Definition 18 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota$. $\lambda V0x \in A_27a$. $\lambda V1s \in (2^{A_27a})$.(ap $(c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0x))$).

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 20 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 21 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V0x$.

Definition 22 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27a}). \lambda V0x$.

Definition 23 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota. (ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Epred_set_2ECARD A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 24 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (15)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal}) \quad (18)$$

Definition 29 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge (p\ V1t2) \wedge (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.(\forall V2z \in A_27a.(((V0x = V1y) \wedge (V1y = V2z)) \Rightarrow (V0x = V2z)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (28)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \quad (29)$$

$$(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1P \in$$

$$(2^{A_27a}). ((p \ (ap \ (c_2Epred_set_2EFINITE \ A_27a) \ V0s)) \Rightarrow (p \ (ap$$

$$(c_2Epred_set_2EFINITE \ A_27a) \ (ap \ (c_2Epred_set_2EGSPEC \ A_27a$$

$$A_27a) \ (\lambda V2x \in A_27a.(ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ 2) \ V2x) \ ($$

$$ap \ (ap \ c_2Ebool_2E_2F_5C \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0s))$$

$$(ap \ V1P \ V2x))))))$$
(30)

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}).$$

$$(\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}).$$

$$((\forall V3x \in A_27a. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \ V3x) \ V2s)) \Rightarrow$$

$$((ap \ V0f \ V3x) = (ap \ V1g \ V3x)))) \Rightarrow ((ap \ (ap \ (c_2Eiterate_2ESum \ A_27a)$$

$$V2s) \ V0f) = (ap \ (ap \ (c_2Eiterate_2ESum \ A_27a) \ V2s) \ V1g))))$$
(31)

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0c \in ty_2Erealax_2Ereal.$$

$$(\forall V1s \in (2^{A_27a}). ((p \ (ap \ (c_2Epred_set_2EFINITE \ A_27a)$$

$$V1s)) \Rightarrow ((ap \ (ap \ (c_2Eiterate_2ESum \ A_27a) \ V1s) \ (\lambda V2n \in A_27a.$$

$$V0c)) = (ap \ (ap \ c_2Erealax_2Ereal_mul \ (ap \ c_2Ereal_2Ereal_of_num$$

$$(ap \ (c_2Epred_set_2ECARD \ A_27a) \ V1s))) \ V0c))))$$
(32)

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow ($$

$$\forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1f \in ((ty_2Erealax_2Ereal^{A_27b})^{A_27a}).$$

$$(\forall V2s \in (2^{A_27a}). (\forall V3t \in (2^{A_27b}). ((p \ (ap \ (c_2Epred_set_2EFINITE$$

$$A_27a) \ V2s)) \wedge (p \ (ap \ (c_2Epred_set_2EFINITE \ A_27b) \ V3t))) \Rightarrow (($$

$$ap \ (ap \ (c_2Eiterate_2ESum \ A_27a) \ V2s) \ (\lambda V4x \in A_27a.(ap \ (ap \ ($$

$$c_2Eiterate_2ESum \ A_27b) \ (ap \ (c_2Epred_set_2EGSPEC \ A_27b \ A_27b)$$

$$(\lambda V5y \in A_27b.(ap \ (ap \ (c_2Epair_2E_2C \ A_27b \ 2) \ V5y) \ (ap \ (ap \ c_2Ebool_2E_2F_5C$$

$$(ap \ (ap \ (c_2Ebool_2EIN \ A_27b) \ V5y) \ V3t))) \ (ap \ (ap \ V0R \ V4x) \ V5y))))))$$

$$(\lambda V6y \in A_27b.(ap \ (ap \ V1f \ V4x) \ V6y)))) = (ap \ (ap \ (c_2Eiterate_2ESum$$

$$A_27b) \ V3t) \ (\lambda V7y \in A_27b.(ap \ (ap \ (c_2Eiterate_2ESum \ A_27a)$$

$$(ap \ (c_2Epred_set_2EGSPEC \ A_27a \ A_27a) \ (\lambda V8x \in A_27a.(ap \ ($$

$$ap \ (c_2Epair_2E_2C \ A_27a \ 2) \ V8x) \ (ap \ (ap \ c_2Ebool_2E_2F_5C \ (ap$$

$$(ap \ (c_2Ebool_2EIN \ A_27a) \ V8x) \ V2s)) \ (ap \ (ap \ V0R \ V8x) \ V7y)))))) \ ($$

$$\lambda V9x \in A_27a.(ap \ (ap \ V1f \ V9x) \ V7y))))))$$
(33)

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\ & (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow ((ap\ c_2Ereal_2Ereal_of_num \\ & (ap\ (c_2Epred_set_2ECARD\ A_27a)\ V0s)) = (ap\ (ap\ (c_2Eiterate_2ESum \\ & A_27a)\ V0s)\ (\lambda V1x \in A_27a.(ap\ c_2Ereal_2Ereal_of_num\ (ap \\ & c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_mul \\ & V0x)\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = V0x)) \end{aligned} \quad (35)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\ & (2^{A_27b}). (\forall V3k \in (ty_2Enum_2Enum^{A_27b}). (((p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27a)\ V1s)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V2t)) \wedge (\forall V4j \in \\ & A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V4j)\ V2t)) \Rightarrow ((ap\ (c_2Epred_set_2ECARD \\ & A_27a)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ (\lambda V5i \in A_27a. \\ & (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V5i)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5i)\ V1s))\ (ap\ (ap\ V0R\ V5i)\ V4j)))))) = \\ & (ap\ V3k\ V4j)))))) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2ESum\ A_27a)\ V1s)\ (\lambda V6i \in \\ & A_27a.(ap\ c_2Ereal_2Ereal_of_num\ (ap\ (c_2Epred_set_2ECARD \\ & A_27b)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27b\ A_27b)\ (\lambda V7j \in A_27b. \\ & (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2)\ V7j)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V7j)\ V2t))\ (ap\ (ap\ V0R\ V6i)\ V7j))))))))) = \\ & (ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ V2t)\ (\lambda V8i \in A_27b.(ap\ c_2Ereal_2Ereal_of_num \\ & (ap\ V3k\ V8i))))))))) \end{aligned}$$