thm_2Eiterate_2ESUM__PARTIAL__PRE (TMYLPXWQY-hzhjPr75M9zeph7FagbCepx9ND)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (6)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (8)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge p$ *of type* $\iota\Rightarrow\iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Definition 17 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 18 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (10)$$

Let $ty\_2Epair\_2Eprod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (11)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (12)$$

Definition 19 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{13}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{15}$$

Definition 20 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 21 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_neg}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal}$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{16}$$

Definition 22 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_add}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 23 *We define* $\mathsf{c\_2Ereal\_2Ereal\_\_sub}$ *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{17}$$

Definition 24 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_mul}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{18}$$

Definition 25 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{19}$$

Definition 26 *We define* c_2Eiterate_2E_2E_2E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 27 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap$ (c_2Emin

Definition 28 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 29 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 30 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 31 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 32 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 33 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 34 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 35 *We define* c_2Eiterate_2ESum *to be* $\lambda A\_27a : \iota.(ap$ (c_2Eiterate_2Eiterate $A\_27a\ ty\_2Erealax$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1c))\ V1c) = V0a))) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \quad (22)$$

Assume the following.

$$\begin{gathered}
(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
(ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V2m \in ty_2Enum_2Enum. \\
(\forall V3n \in ty_2Enum_2Enum.((ap\ (ap\ (\mathsf{c_2Eiterate_2ESum}\ ty_2Enum_2Enum) \\
(ap\ (ap\ \mathsf{c_2Eiterate_2E_2E_2E}\ V2m)\ V3n))\ (\lambda V4k \in ty_2Enum_2Enum. \\
(ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul}\ (ap\ V0f\ V4k))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub} \\
(ap\ V1g\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V4k)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V1g \\
V4k))))) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ ty_2Erealax_2Ereal)\ (ap \\
(ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V2m)\ V3n))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub} \\
(ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub}\ (ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul}\ (\\
ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V3n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V1g \\
(ap\ (ap\ c_2Earithmetic_2E_2B\ V3n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO}))))))\ (ap\ (\\
ap\ \mathsf{c_2Erealax_2Ereal__mul}\ (ap\ V0f\ V2m))\ (ap\ V1g\ V2m))))\ (ap\ (ap\ (\\
\mathsf{c_2Eiterate_2ESum}\ ty_2Enum_2Enum)\ (ap\ (ap\ \mathsf{c_2Eiterate_2E_2E_2E} \\
V2m)\ V3n))\ (\lambda V5k \in ty_2Enum_2Enum.(ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul} \\
(ap\ V1g\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V5k)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ (ap \\
\mathsf{c_2Ereal_2Ereal__sub}\ (ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V5k) \\
(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))) \\
(ap\ V0f\ V5k)))))))\ (ap\ c_2Ereal_2Ereal__of__num\ \mathsf{c_2Enum_2E0})))))))
\end{gathered} \tag{23}$$

Theorem 1

$$\begin{gathered}
(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
(ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V2m \in ty_2Enum_2Enum. \\
(\forall V3n \in ty_2Enum_2Enum.((ap\ (ap\ (\mathsf{c_2Eiterate_2ESum}\ ty_2Enum_2Enum) \\
(ap\ (ap\ \mathsf{c_2Eiterate_2E_2E_2E}\ V2m)\ V3n))\ (\lambda V4k \in ty_2Enum_2Enum. \\
(ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul}\ (ap\ V0f\ V4k))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub} \\
(ap\ V1g\ V4k))\ (ap\ V1g\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V4k)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))))) = (\\
ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ ty_2Erealax_2Ereal)\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D} \\
V2m)\ V3n))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub}\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub} \\
(ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul}\ (ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
V3n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
\mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V1g\ V3n)))\ (ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul} \\
(ap\ V0f\ V2m))\ (ap\ V1g\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V2m)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))))\ (ap \\
(ap\ (\mathsf{c_2Eiterate_2ESum}\ ty_2Enum_2Enum)\ (ap\ (ap\ \mathsf{c_2Eiterate_2E_2E_2E} \\
V2m)\ V3n))\ (\lambda V5k \in ty_2Enum_2Enum.(ap\ (ap\ \mathsf{c_2Erealax_2Ereal__mul} \\
(ap\ V1g\ V5k))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub}\ (ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
V5k)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1} \\
\mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V0f\ V5k)))))))\ (ap\ c_2Ereal_2Ereal__of__num \\
\mathsf{c_2Enum_2E0})))))))
\end{gathered}$$