

thm_2Eiterate_2ESUM__SUBSET__SIMPLE
(TMGC-
nSY3FvXdTtTdzYLYpJXKXU8oBZsNuKP)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (5)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (7)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V0t))))^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}$

Definition 11 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin_2E_3D_3D_3E) op)$

Definition 12 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap (c_2Emin_2E_3D_3D_3E) V0t) c_2Ebool_2E_2F))$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_3D_3D_3E) (c_2Epair_2EABS_prod\ A_27a\ A_27b\ x\ y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (9)$$

Definition 16 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1x \in A_27a.\lambda V2y \in A_27b.(ap (c_2Eiterate_2Eneutral) op x y)$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_2F) t1 t2))))$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V0t))))^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}$

Definition 19 We define $c_2\text{Epred_set_2EINSERT}$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 20 We define $c_2\text{Epred_set_2EEMPTY}$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 21 We define $c_2\text{Epred_set_2EFINITE}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 22 We define $c_2\text{Eiterate_2EITSET}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V1g \in (A_27b^{A_27b}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 23 We define $c_2\text{Eiterate_2Eiterate}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1g \in (A_27b^{A_27b}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 24 We define $c_2\text{Eiterate_2ESum}$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax_2Ereal_of_num))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \quad (12)$$

Definition 25 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (14)$$

Definition 26 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 27 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 28 We define $c_2\text{Epred_set_2EDIFF}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 29 We define $c_2\text{Epred_set_2ESUBSET}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2^{A_27a})))$.

Assume the following.

$$\text{True} \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3)))))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow \text{False}) \Rightarrow \neg(p \ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p \ V0t) \Rightarrow ((p \ V0t) \Rightarrow \text{False}))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge ((p \ V0t) \Rightarrow \text{False}) \Leftrightarrow \neg(p \ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.(\neg(\neg(p \ V0t)) \Leftrightarrow (p \ V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (21)$$

Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow \neg(p \ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (24)$$

Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p \ V0P) \wedge (\forall V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))))) \quad (25)$$

Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \vee (\exists V4x \in A.27a.(p \ (ap \ V1Q \ V4x))))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A-27b})^{A-27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))))) \Leftrightarrow (\exists V3f \in (A.27b^{A-27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0u \in (2^{A-27a}). (\forall V1v \in (2^{A-27a}). (\forall V2f \in (ty_2Erealax_2Ereal^{A-27a}). (((p (ap (c_2Epred_set_2EFINITE A.27a) V0u)) \wedge ((p (ap (c_2Epred_set_2EFINITE A.27a) V1v)) \wedge ((\forall V3x \in A.27a. ((p (ap (ap (c_2Ebool_2EIN A.27a) V3x) (ap (ap (c_2Epred_set_2EDIFF A.27a) V0u) V1v)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap V2f V3x)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))) \wedge (\forall V4x \in A.27a. ((p (ap (ap (c_2Ebool_2EIN A.27a) V4x) (ap (ap (c_2Epred_set_2EDIFF A.27a) V1v) V0u)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) (ap V2f V4x)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap (c_2Eiterate_2ESum A.27a) V0u) V2f)) (ap (ap (c_2Eiterate_2ESum A.27a) V1v) V2f)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\\ & ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\ & (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27a)\ V1t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q)) \vee (\neg(p\ V2r))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge ((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{46}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0u \in (2^{A_{27a}}). (\forall V1v \in \\
& (2^{A_{27a}}). (\forall V2f \in (\text{ty_2Erealax_2Ereal}^{A_{27a}}). (((p (ap \\
& (\text{c_2Epred_set_2EFINITE } A_{27a}) V1v)) \wedge ((p (ap (ap (\text{c_2Epred_set_2ESUBSET} \\
& A_{27a}) V0u) V1v)) \wedge (\forall V3x \in A_{27a}. ((p (ap (ap (\text{c_2Ebool_2EIN} \\
& A_{27a}) V3x) (ap (ap (\text{c_2Epred_set_2EDIFF } A_{27a}) V1v) V0u))) \Rightarrow (\\
& p (ap (ap (\text{c_2Ereal_2Ereal_lte } (ap \text{c_2Ereal_2Ereal_of_num } \text{c_2Enum_2E0})) \\
& (ap V2f V3x)))))) \Rightarrow (p (ap (ap (\text{c_2Ereal_2Ereal_lte } (ap (ap (\text{c_2Eiterate_2ESum} \\
& A_{27a}) V0u) V2f)) (ap (ap (\text{c_2Eiterate_2ESum } A_{27a}) V1v) V2f))))))
\end{aligned}$$