

thm_2Eiterate_2ESUM_SWAP (TMZGK- mATT9YX77VoRAVJRrn3LhyNmzK3byf)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ecombin_2ES` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. (\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Definition 4 We define `c_2Ecombin_2EC` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. (\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Definition 5 We define `c_2Ecombin_2EK` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. (\lambda V0x \in A.27a. (\lambda V1y \in A.27b.V0x))$

Definition 6 We define `c_2Ecombin_2EI` to be $\lambda A. \lambda a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A.27a (A.27a^{A.27a})) A.27a))$

Definition 7 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A.27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A.27a}))))$

Definition 8 We define `c_2Ecombin_2Eo` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda A. \lambda c : \iota. \lambda V0f \in (A.27b^{A.27c}). \lambda V1g$

Definition 9 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 11 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Definition 12 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define `c_2Ebool_2ECOND` to be $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A.27a. (\lambda V2t2 \in A.27a. ($

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod A0 A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (3)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (4)$$

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ t$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (5)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 15 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 16 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Definition 18 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (c_2Emin$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2).(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 20 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 21 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (13)$$

Definition 22 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27a}). \lambda V1x \in A_27a. (ap\ (c_2Eiterate_2Eiterate\ x\ op))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (V0t1\ V1t2))))$

Definition 24 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (V0x\ s))$

Definition 25 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 26 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (V0s))$

Definition 27 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V1x \in A_27a. (ap\ (c_2Eiterate_2Eiterate\ x\ f))$

Definition 28 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27a}). \lambda V1x \in A_27a. (ap\ (c_2Eiterate_2Eiterate\ x\ op))$

Definition 29 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota. (ap\ (c_2Eiterate_2Eiterate\ A_27a\ ty_2Erealax))$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2)))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& \quad (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& \quad (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& \quad V5y_27)))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\
& \quad A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f)))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). ((ap\ (ap\ (c_2Eiterate_2ESum \\
& \quad A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a))\ V0f) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \wedge (\forall V1x \in A_27b. (\forall V2f \in (ty_2Erealax_2Ereal^{A_27b}). \\
& \quad (\forall V3s \in (2^{A_27b}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b) \\
& \quad V3s)) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A_27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal) \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V1x)\ V3s))\ (ap\ (ap\ (c_2Eiterate_2ESum \\
& \quad A_27b)\ V3s)\ V2f))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ V2f\ V1x))\ (\\
& \quad ap\ (ap\ (c_2Eiterate_2ESum\ A_27b)\ V3s)\ V2f)))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). \\
& \quad (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2ESum \\
& \quad A_27a)\ V2s)\ (\lambda V3x \in A_27a. (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap \\
& \quad V0f\ V3x))\ (ap\ V1g\ V3x)))) = (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap \\
& \quad (c_2Eiterate_2ESum\ A_27a)\ V2s)\ V0f))\ (ap\ (ap\ (c_2Eiterate_2ESum \\
& \quad A_27a)\ V2s)\ V1g)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((ap\ (ap \\
& \quad (c_2Eiterate_2ESum\ A_27a)\ V0s)\ (\lambda V1n \in A_27a. (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).((\\
& (p\ (ap\ V0P\ (c.2Epred_set.2EEMPTY\ A.27a))) \wedge (\forall V1s \in (2^{A-27a}). \\
& ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& (\forall V2e \in A.27a.((\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2e)\ V1s))) \Rightarrow \\
& (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V2e)\ V1s)))))) \Rightarrow \\
& (\forall V3s \in (2^{A-27a}).((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\
& V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\
& \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (47)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \quad \forall V0f \in ((ty.2Erealax.2Ereal^{A.27b})^{A.27a}). (\forall V1s \in \\ & \quad (2^{A.27a}). (\forall V2t \in (2^{A.27b}). (((p (ap (c.2Epred_set.2EFINITE \\ & \quad A.27a) V1s)) \wedge (p (ap (c.2Epred_set.2EFINITE A.27b) V2t)))) \Rightarrow ((\\ & \quad ap (ap (c.2Eiterate.2ESum A.27a) V1s) (\lambda V3i \in A.27a. (ap (ap (\\ & \quad c.2Eiterate.2ESum A.27b) V2t) (ap V0f V3i)))) = (ap (ap (c.2Eiterate.2ESum \\ & \quad A.27b) V2t) (\lambda V4j \in A.27b. (ap (ap (c.2Eiterate.2ESum A.27a) \\ & \quad V1s) (\lambda V5i \in A.27a. (ap (ap V0f V5i) V4j)))))))))) \end{aligned}$$