

thm_2Eiterate_2ESUP__UNION (TMLuBTPH- FUsgRnTtezJRsmYXD5Zr6TwNWFkx)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \ x) \text{ then } (the \ (\lambda x. x \in A \wedge P \ x) \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap \ V0P \ (ap \ (c_2Emin_2E_40 \ A \ V0P))))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap \ (ap \ (c_2Emin_2E_3D \ (2^{A-27a}) \ V0P))))$

Definition 6 We define `c_2Ebool_2EF` to be $(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t)).$

Definition 7 We define `c_2Epred__set_2EEMPTY` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. c_2Ebool_2EF).$

Let `ty_2Ehreal_2Ehreal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Ehreal_2Ehreal \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod \ A0 \ A1) \tag{2}$$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Erealax_2Ereal \tag{3}$$

Let `c_2Erealax_2Ereal__REP__CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)}) \ ty_2Erealax_2Ereal) \tag{4}$$

Definition 8 We define `c_2Erealax_2Ereal__REP` to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap \ (c_2Emin_2E_40 \ (ty_2Erealax_2Ereal \ V0a)))$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (\\ & ap V1Q V4x)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (\\ & 2^{A.27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in \\ & A.27a.(p (ap V1P V3x))) \vee (p V0Q)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ & (p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge \\ & (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((\neg(V0s = (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal)))) \wedge (\exists V1b \in ty_2Erealax_2Ereal.(\forall V2x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal \\
& V2x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V2x) V1b)))))) \Rightarrow ((\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal \\
& V3x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V3x) (ap c_2Ereal_2Esup \\
& V0s)))))) \wedge (\forall V4b \in ty_2Erealax_2Ereal.((\forall V5x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V0s)) \Rightarrow (p (\\
& ap (ap c_2Ereal_2Ereal_lte V5x) V4b)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad (ap c_2Ereal_2Esup V0s) V4b)))))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1b \in ty_2Erealax_2Ereal. \\
& (\forall V2c \in ty_2Erealax_2Ereal.((\forall V3x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V0s)) \Rightarrow (p (\\
& ap (ap c_2Ereal_2Ereal_lte V3x) V2c)))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V1b) V2c)))) \Rightarrow ((ap c_2Ereal_2Esup V0s) = V1b))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1s \in \\
& \quad (2^{A_27a}).(\forall V2t \in (2^{A_27a}).((\forall V3x \in A_27a.((p (\\
& ap (ap (c_2Ebool_2EIN A_27a) V3x) (ap (ap (c_2Epred_set_2EUNION \\
& \quad A_27a) V1s) V2t))) \Rightarrow (p (ap V0P V3x)))) \Leftrightarrow ((\forall V4x \in A_27a.((p \\
& (ap (ap (c_2Ebool_2EIN A_27a) V4x) V1s)) \Rightarrow (p (ap V0P V4x)))) \wedge (\forall V5x \in \\
& A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V5x) V2t)) \Rightarrow (p (ap V0P V5x))))))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (\\
& \quad ap c_2Ereal_2Ereal_lte V0x) V2z))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap (ap c_2Ereal_2Ereal_lte V1x) V2y)) V0z)) \Leftrightarrow ((p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V1x) V0z)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V2y) V0z))))))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{24}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (35)$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & (((\neg(V0s = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))) \wedge ((\\ & \neg(V1t = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))) \wedge ((\exists V2b \in \\ & ty_2Erealax_2Ereal.(\forall V3x \in ty_2Erealax_2Ereal.((p (ap \\ (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\ V3x) V2b)))))) \wedge (\exists V4c \in ty_2Erealax_2Ereal.(\forall V5x \in \\ ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ V5x) V1t)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V5x) V4c))))))) \Rightarrow (\\ (ap c_2Ereal_2Esup (ap (ap (c_2Epred_set_2EUNION ty_2Erealax_2Ereal) \\ V0s) V1t)) = (ap (ap c_2Ereal_2Emax (ap c_2Ereal_2Esup V0s)) (ap \\ c_2Ereal_2Esup V1t)))))) \end{aligned}$$