

# thm\_2Eiterate\_2ESUP\_\_UNIQUE

(TMYax5p85hMiEWSXScp1Gc6MCzK1teAdfkt)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow P \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Ehreal\_2Ehreal} \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \tag{2}$$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Erealax\_2Ereal} \tag{3}$$

Let `c_2Erealax_2Ereal__REP__CLASS` :  $\iota$  be given. Assume the following.

$$\text{c\_2Erealax\_2Ereal\___REP\___CLASS} \in ((2^{(\text{ty\_2Epair\_2Eprod } \text{ty\_2Ehreal\_2Ehreal } \text{ty\_2Ehreal\_2Ehreal}) \text{ty\_2Erealax\_2Ereal}})) \tag{4}$$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P \ x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Erealax_2Ereal__REP` to be  $\lambda V0a \in \text{ty\_2Erealax\_2Ereal}. (\text{ap } (\text{c\_2Emin\_2E\_40 } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))))$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 11** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal))))$

**Definition 13** We define  $c\_2Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal))$

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_7E)$ .

**Definition 15** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2))))$

**Definition 16** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (6)$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ x\ y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (7)$$

**Definition 18** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Epair\_2E\_2C\ x\ s))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ (ap \ V0f \ V1v)))))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((\forall V2b \in ty\_2Erealax\_2Ereal.((\forall V3x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3x) V0s)) \Rightarrow (p ( \\
& ap (ap c\_2Ereal\_2Ereal\_lte V3x) V2b)))) \Leftrightarrow (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4x) V1t)) \Rightarrow (p ( \\
& ap (ap c\_2Ereal\_2Ereal\_lte V4x) V2b)))))) \Rightarrow ((ap c\_2Ereal\_2Esup \\
& V0s) = (ap c\_2Ereal\_2Esup V1t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal.((ap c\_2Ereal\_2Esup (ap (ap \\
& (c\_2Epred\_set\_2EINSERT ty\_2Erealax\_2Ereal) V0a) (c\_2Epred\_set\_2EEMPTY \\
& ty\_2Erealax\_2Ereal))) = V0a))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg (p (ap (ap \\
& (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EEMPTY A\_27a))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.(\forall V2s \in (2^{A\_27a}).((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\
& V0x) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V2s)))))))))
\end{aligned} \tag{24}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1b \in ty\_2Erealax\_2Ereal. \\
& ((\forall V2c \in ty\_2Erealax\_2Ereal.((\forall V3x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3x) V0s)) \Rightarrow (p ( \\
& ap (ap c\_2Ereal\_2Ereal\_lte V3x) V2c)))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1b) V2c)))) \Rightarrow ((ap c\_2Ereal\_2Esup V0s) = V1b))))))
\end{aligned}$$