thm_2Eiterate_2Ereal__INFINITE (TMWoD-VbB8j6WZZ69f9ddqRLZFdUCNPkxQB7)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{5}$$

Definition 7 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 10 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{6}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{7}$$

Definition 13 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 14 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 15 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 17 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{11}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{13}$$

Assume the following.

$$((\forall V0a \in \mathit{ty\_2Erealax\_2Ereal}.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V1x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V1x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0a)\ V1x))))))))\land((\forall V2a \in \mathit{ty\_2Erealax\_2Ereal}.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V3x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V3x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V2a)\ V3x))))))))\land((\forall V4b \in \mathit{ty\_2Erealax\_2Ereal}.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V5x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V5x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V5x)\ V4b))))))))\land((\forall V6b \in \mathit{ty\_2Erealax\_2Ereal}.(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V7x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V7x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V7x)\ V6b))))))))\land((\forall V8a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V9b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V10x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V10x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V8a)\ V10x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V10x)\ V9b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V9b)\ V8a)))))\land((\forall V11a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V12b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V13x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V13x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V11a)\ V13x))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V13x)\ V12b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V12b)\ V11a)))))\land((\forall V14a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V15b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V16x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V16x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V14a)\ V16x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V16x)\ V15b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V15b)\ V14a)))))\land(\forall V17a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V18b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ \mathit{ty\_2Erealax\_2Ereal})\ (ap\ (\mathit{c\_2Epred\_\_set\_2EGSPEC}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ (\lambda V19x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ 2)\ V19x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V17a)\ V19x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V19x)\ V18b)))))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V18b)\ V17a)))))))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1t)))))) \tag{16}$$

Theorem 1

$$(\neg(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Erealax\_2Ereal)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal))))$$