

thm\_2Elbtree\_2ENd\_\_11  
(TMcymn6gf7gh9AV66DKkdJUfjJwBnePkcCa)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

**Definition 5** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A_{27a} : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_{27a}. (\lambda V2t2 \in A_{27a}. (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\lambda V3t3 \in A_{27a}. V3t3))$

Let `ty_2Eone_2Eone` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Eone\_2Eone} \tag{1}$$

**Definition 9** We define `c_2Eone_2Eone` to be  $(\text{ap } (\text{c_2Emin_2E_40 } \text{ty\_2Eone\_2Eone}) (\lambda V0x \in \text{ty\_2Eone\_2Eone}. V0x))$

**Definition 10** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\lambda V1t1 \in 2. V1t1)) (\lambda V2t2 \in 2. V2t2))$

Let `ty_2Esum_2Esum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Esum\_2Esum } A0 \ A1) \tag{2}$$

Let `c_2Esum_2EABS_sum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \text{c\_2Esum\_2EABS\_sum } A_{27a} \ A_{27b} \in ((\text{ty\_2Esum\_2Esum } A_{27a} \ A_{27b})^{((2^{A_{27b}})^{A_{27a}})^2}) \tag{3}$$

**Definition 11** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 12** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c\_2Eoption\_2ENONE A\_27a))$

**Definition 13** We define  $c\_2Elbtree\_2ELfrep$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0l \in A\_27a. (c\_2Eoption\_2ENONE A\_27a (c\_2Elbtree\_2ELfrep A\_27a V0l)))$

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A\_27a V0P))))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (6)$$

**Definition 15** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_Esum\_2EABS A\_27a A\_27b V0e))$

**Definition 16** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a V0x))$

Let  $c\_2Elist\_2Elist\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Elist\_2Elist\_CASE A\_27a A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (7)$$

**Definition 17** We define  $c\_2Elbtree\_2ENDrep$  to be  $\lambda A\_27a : \iota. \lambda V0a \in A\_27a. \lambda V1t1 \in ((ty\_2Eoption\_2Eoption A\_27a V0a))$

**Definition 18** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_3F A\_27a V2t))))))$

**Definition 19** We define  $c\_2Elbtree\_2Eis\_lbtree$  to be  $\lambda A\_27a : \iota. \lambda V0t \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})$

Let  $ty\_2Elbtree\_2Elbtree : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elbtree\_2Elbtree A0) \quad (8)$$

Let  $c\_2Elbtree\_2Elbtree\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_rep A\_27a \in (((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Elist\_2Elist 2)})^{(ty\_2Elbtree\_2Elbtree A\_27a)}) \quad (9)$$

Let  $c\_2Elbtree\_2Elbtree\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_abs A\_27a \in ((ty\_2Elbtree\_2Elbtree A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Elist\_2Elist 2)}}) \quad (10)$$

**Definition 20** We define  $c\_2Elbtree\_2ENd$  to be  $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in (ty\_2Elbtree\_2Elbtree$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (11)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \quad (29)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in \\
& A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ ( \\
& ap\ V0P\ V1a))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\
& (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Elbtree\_2Elbtree \\
& A\_27a). ((ap\ (c\_2Elbtree\_2Elbtree\_abs\ A\_27a)\ (ap\ (c\_2Elbtree\_2Elbtree\_rep \\
& A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)}). \\
& ((p\ (ap\ (c\_2Elbtree\_2Eis\_lbtree\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Elbtree\_2Elbtree\_rep \\
& A\_27a)\ (ap\ (c\_2Elbtree\_2Elbtree\_abs\ A\_27a)\ V1r)) = V1r))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0v \in A\_27b. (\forall V1f \in ((A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). \\
& ((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b)\ (c\_2Elist\_2ENIL \\
& A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A\_27a. (\forall V3a1 \in \\
& (ty\_2Elist\_2Elist\ A\_27a). (\forall V4v \in A\_27b. (\forall V5f \in ( \\
& (A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE \\
& A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\
& (ap\ (ap\ V5f\ V2a0)\ V3a1)))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (((ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME \\
& A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y))))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (51)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0a1 \in A.27a.(\forall V1t1 \in \\ & (ty\_2Elbtree\_2Elbtree \ A.27a).(\forall V2u1 \in (ty\_2Elbtree\_2Elbtree \\ & A.27a).(\forall V3a2 \in A.27a.(\forall V4t2 \in (ty\_2Elbtree\_2Elbtree \\ & A.27a).(\forall V5u2 \in (ty\_2Elbtree\_2Elbtree \ A.27a).(((ap \ (ap \\ & (ap \ (c\_2Elbtree\_2ENd \ A.27a) \ V0a1) \ V1t1) \ V2u1) = (ap \ (ap \ (ap \ (c\_2Elbtree\_2ENd \\ & A.27a) \ V3a2) \ V4t2) \ V5u2)) \Leftrightarrow ((V0a1 = V3a2) \wedge ((V1t1 = V4t2) \wedge (V2u1 = \\ & V5u2)))))))))) \end{aligned}$$