

# thm\_2Elbtree\_2Edpth\_\_rules (TMd- pyEKuWHticKzTyhK4WvzBQtnD1gPF1H6)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (6)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (7)$$

Let  $ty\_2Elbtree\_2Elbtree : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elbtree\_2Elbtree\ A0) \quad (8)$$

Let  $c\_2Elbtree\_2Elbtree\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elbtree\_2Elbtree\_rep\ A.27a \in \\ ((ty\_2Eoption\_2Eoption\ A.27a)^{(ty\_2Elist\_2Elist\ 2)}(ty\_2Elbtree\_2Elbtree\ A.27a)) \quad (9)$$

**Definition 9** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.))$

**Definition 10** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (10)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (11)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Esum\_2EABS\_sum\ A.27a\ A.27b \in ((ty\_2Esum\_2Esum\ A.27a\ A.27b)^{((2^{A.27b})^{A.27a})^2}) \quad (12)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0e \in A.27a.(ap\ (c\_2Esum\_2EABS\_sum\ A.27a\ A.27b))$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A.27a \in \\ ((ty\_2Eoption\_2Eoption\ A.27a)^{(ty\_2Esum\_2Esum\ A.27a\ ty\_2Eone\_2Eone)}) \quad (13)$$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A.27a : \iota.\lambda V0x \in A.27a.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A.27a\ V0x))$

Let  $c\_2Elist\_2Elist\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_CASE \\ & A\_27a\ A\_27b \in (((A\_27b((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}))_{A\_27b}(ty\_2Elist\_2Elist\ A\_27a)) \end{aligned} \quad (14)$$

**Definition 14** We define  $c\_2Elbtree\_2ENdrep$  to be  $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in ((ty\_2Eoption\_2Eop$   
Let  $c\_2Elbtree\_2Elbtree\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_abs\ A\_27a \in \\ & ((ty\_2Elbtree\_2Elbtree\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)}}) \end{aligned} \quad (15)$$

**Definition 15** We define  $c\_2Elbtree\_2ENd$  to be  $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in (ty\_2Elbtree\_2Elbtree$

**Definition 16** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E$

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 18** We define  $c\_2Elbtree\_2Edpeth$  to be  $\lambda A\_27a : \iota.(\lambda V0a0 \in A\_27a.(\lambda V1a1 \in (ty\_2Elbtree\_2Elb$   
Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow \\ & ((p\ V1y) \wedge (p\ V3w)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow \\ & ((p\ V1y) \vee (p\ V3w)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow \\ & ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ ( \\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (21)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0x \in A\_27a. (\forall V1t1 \in \\ & (ty\_2Elbtree\_2Elbtree\ A\_27a). (\forall V2t2 \in (ty\_2Elbtree\_2Elbtree \\ & A\_27a). (p\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2Edepth\ A\_27a)\ V0x)\ (ap\ (ap\ (ap \\ & (c\_2Elbtree\_2ENd\ A\_27a)\ V0x)\ V1t1)\ V2t2))\ c\_2Enum\_2E0)))))) \wedge ( \\ & (\forall V3m \in ty\_2Enum\_2Enum. (\forall V4x \in A\_27a. (\forall V5a \in \\ & A\_27a. (\forall V6t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a). (\forall V7t2 \in \\ & (ty\_2Elbtree\_2Elbtree\ A\_27a). ((p\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2Edepth \\ & A\_27a)\ V4x)\ V6t1)\ V3m)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2Edepth \\ & V4x)\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2ENd\ A\_27a)\ V5a)\ V6t1)\ V7t2))\ (ap\ c\_2Enum\_2ESUC \\ & V3m)))))))))) \wedge (\forall V8m \in ty\_2Enum\_2Enum. (\forall V9x \in A\_27a. \\ & (\forall V10a \in A\_27a. (\forall V11t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a). \\ & (\forall V12t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a). ((p\ (ap\ (ap\ (ap\ ( \\ & c\_2Elbtree\_2Edepth\ A\_27a)\ V9x)\ V12t2)\ V8m)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2Edepth \\ & A\_27a)\ V9x)\ (ap\ (ap\ (ap\ (c\_2Elbtree\_2ENd\ A\_27a)\ V10a)\ V11t1)\ V12t2)) \\ & (ap\ c\_2Enum\_2ESUC\ V8m))))))))))))) \end{aligned}$$