thm_2Elbtree_2Edepth__strongind (TMa29bJ4QveX3avNwnXahn9weUhGw9ZKwyM)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (6)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (7)$$

Let $ty\_2Elbtree\_2Elbtree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elbtree\_2Elbtree\ A0) \quad (8)$$

Let $c\_2Elbtree\_2Elbtree\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})^{(ty\_2Elbtree\_2Elbtree\ A\_27a)}) \quad (9)$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (10)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (11)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (12)$$

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (13)$$

Definition 13 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Elist\_2Elist\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Definition 14 *We define* $\mathsf{c\_2Elbtree\_2ENdrep}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in ((ty\_2Eoption\_2Eop$

Let $c\_2Elbtree\_2Elbtree\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_abs\ A\_27a \in ((ty\_2Elbtree\_2Elbtree\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})}) \tag{15}$$

Definition 15 *We define* $\mathsf{c\_2Elbtree\_2ENd}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in (ty\_2Elbtree\_2Elbtree$

Definition 16 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 17 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 18 *We define* $\mathsf{c\_2Elbtree\_2Edepth}$ *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in A\_27a.(\lambda V1a1 \in (ty\_2Elbtree\_2El$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{18}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{19}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{21}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0depth\_27 \in (((2^{ty\_2Enum\_2Enum})^{(ty\_2Elbtree\_2Elbtree\ A\_27a)})^{A\_27a}). (((\forall V1x \in A\_27a.(\forall V2t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a). (\forall V3t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(p\ (ap\ (ap\ (ap\ V0depth\_27\ V1x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V1x)\ V2t1)\ V3t2))\ \mathsf{c\_2Enum\_2E0})))))\wedge ((\forall V4m \in ty\_2Enum\_2Enum.(\forall V5x \in A\_27a.(\forall V6a \in A\_27a.(\forall V7t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V8t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Edepth}\ A\_27a)\ V5x)\ V7t1)\ V4m))\wedge(p\ (ap\ (ap\ (ap\ V0depth\_27\ V5x)\ V7t1)\ V4m)))\Rightarrow (p\ (ap\ (ap\ (ap\ V0depth\_27\ V5x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V6a)\ V7t1)\ V8t2))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V4m)))))))))\wedge(\forall V9m \in ty\_2Enum\_2Enum.(\forall V10x \in A\_27a.(\forall V11a \in A\_27a.(\forall V12t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V13t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Edepth}\ A\_27a)\ V10x)\ V13t2)\ V9m))\wedge(p\ (ap\ (ap\ (ap\ V0depth\_27\ V10x)\ V13t2)\ V9m)))\Rightarrow(p\ (ap\ (ap\ (ap\ V0depth\_27\ V10x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V11a)\ V12t1)\ V13t2))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V9m)))))))))))\Rightarrow(\forall V14a0 \in A\_27a. (\forall V15a1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V16a2 \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Edepth}\ A\_27a)\ V14a0)\ V15a1)\ V16a2))\Rightarrow(p\ (ap\ (ap\ (ap\ V0depth\_27\ V14a0)\ V15a1)\ V16a2))))))))$$