thm_2Elbtree_2Eexists__bf__flatten (TMVxUC6V782Lgd78bTJN5es4fb3Djjvw5HJ)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if } (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{1}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let $ty\_2Elbtree\_2Elbtree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elbtree\_2Elbtree\ A0) \tag{3}$$

Let $c\_2Elbtree\_2Elbtree\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})^{(ty\_2Elbtree\_2Elbtree\ A\_27a)}) \tag{4}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{5}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{6}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{7}$$

Definition 9 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{8}$$

Definition 10 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Let $c\_2Elist\_2Elist\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Definition 11 *We define* c_2Elbtree_2ENdrep *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in ((ty\_2Eoption\_2Eop$

Let $c\_2Elbtree\_2Elbtree\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_abs\ A\_27a \in ((ty\_2Elbtree\_2Elbtree\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})}) \tag{10}$$

Definition 12 *We define* c_2Elbtree_2ENd *to be* $\lambda A_27a : \iota.\lambda V0a \in A_27a.\lambda V1t1 \in (ty_2Elbtree_2Elbtree\ $

Definition 13 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Elbtree_2Emem *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in A\_27a.(\lambda V1a1 \in (ty\_2Elbtree\_2Elb$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{11}$$

Let $c\_2Elbtree\_2Ebf\_\_flatten : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a))}) \tag{12}$$

Definition 16 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 18 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 19 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 20 *We define* c_2Elbtree_2ELfrep *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0l \in A\_27a.(\mathsf{c\_2Eoption\_2ENON}$

Definition 21 *We define* c_2Elbtree_2ELf *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Elbtree\_2Elbtree\_\_abs\ A\_27a)\ (\mathsf{c\_2Elbtre}$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{13}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{14}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{15}$$

Definition 22 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{16}$$

Let $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{17}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{18}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{19}$$

Definition 23 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{20}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{21}$$

Definition 24 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 25 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{22}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{23}$$

Definition 26 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{24}$$

Definition 27 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 28 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{25}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{26}$$

Definition 29 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

Definition 30 *We define* c_2Ellist_2Eexists *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(\lambda V1a0 \in (ty\_2Ellist\_2Ellis$

Assume the following.

$$True \tag{27}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{31}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{38}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{39}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a) V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in (ty\_2Elbtree\_2Elbtree\ A\_27a).((V0t = (\mathsf{c\_2Elbtree\_2ELf}\ A\_27a))\vee(\exists V1a \in A\_27a. (\exists V2t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\exists V3t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(V0t = (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd} A\_27a)\ V1a)\ V2t1)\ V3t2))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0a \in A\_27a.(\forall V1t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V2t2 \in (ty\_2Elbtree\_2Elbtree A\_27a).(\neg((\mathsf{c\_2Elbtree\_2ELf}\ A\_27a) = (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd} A\_27a)\ V0a)\ V1t1)\ V2t2)))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0a \in A\_27a.(\forall V1b \in A\_27a.(\forall V2t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V3t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem} A\_27a)\ V0a)\ (\mathsf{c\_2Elbtree\_2ELf}\ A\_27a)))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem} A\_27a)\ V0a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V1b)\ V2t1)\ V3t2)))\Leftrightarrow ((V0a = V1b)\vee((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V0a)\ V2t1))\vee(p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V0a)\ V3t2)))))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ (c\_2Elist\_2ENIL\ (ty\_2Elbtree\_2Elbtree\ A\_27a))) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \wedge ((\forall V0ts \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).((ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (\mathsf{c\_2Elbtree\_2ELf}\ A\_27a))\ V0ts)) = (ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ V0ts))) \wedge (\forall V1a \in A\_27a.(\forall V2t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V3t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V4ts \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).((ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V1a)\ V2t1)\ V3t2))\ V4ts)) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V1a)\ (ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ V4ts)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ V2t1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ V3t2)\ (c\_2Elist\_2ENIL\ (ty\_2Elbtree\_2Elbtree\ A\_27a)))))))))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).(((ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ V0l) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (\mathsf{c\_2Elbtree\_2ELf}\ A\_27a)))\ V0l)))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l2 \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).(\forall V1l1 \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (\mathsf{c\_2Elbtree\_2ELf}\ A\_27a)))\ V1l1)) \Rightarrow ((ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ V1l1)\ V0l2)) = (ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ V0l2))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ V1l)) \Rightarrow (\exists V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V3x \in A\_27a.(\exists V4l2 \in (ty\_2Elist\_2Elist\ A\_27a).((V1l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3x)\ V4l2))) \wedge ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ 2)\ \mathsf{c\_2Ebool\_2E\_7E})\ V0P))\ V2l1)) \wedge (p\ (ap\ V0P\ V3x)))))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)))))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow False)) \wedge (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \vee (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V1P)\ V3t)))))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))\ V2l3))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ V1l1)) \vee (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ V2l2))))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((\neg(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V1l))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ 2)\ \mathsf{c\_2Ebool\_2E\_7E})\ V0P))\ V1l))))) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((\neg((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (\neg((\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a.(\forall V1t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h2 \in A\_27a.(\forall V3t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h1)\ V1t1) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((V0h1 = V2h2) \wedge (V1t1 = V3t2)))))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1exists\_27 \in (2^{(ty\_2Ellist\_2Ellist\ A\_27a)}).(((\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ V0P\ V2h)) \Rightarrow (p\ (ap\ V1exists\_27\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h)\ V3t)))))) \wedge (\forall V4h \in A\_27a.(\forall V5t \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ V1exists\_27\ V5t)) \Rightarrow (p\ (ap\ V1exists\_27\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V4h)\ V5t))))))) \Rightarrow (\forall V6a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2Eexists}\ A\_27a)\ V0P)\ V6a0)) \Rightarrow (p\ (ap\ V1exists\_27\ V6a0))))))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg (p\ V1q)) \vee (\neg (p\ V2r)))) \wedge (((p\ V1q) \vee (\neg (p\ V0p))) \wedge ((p\ V2r) \vee (\neg (p\ V0p))))))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg (p\ V1q))) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg (p\ V0p))))))))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((\neg (p\ V1q)) \vee ((p\ V2r) \vee (\neg (p\ V0p))))))))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg (p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg (p\ V0p))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.((\neg (\neg (p\ V0p))) \Rightarrow (p\ V0p))) \tag{71}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1tlist \in (ty\_2Elist\_2Elist\ (ty\_2Elbtree\_2Elbtree\ A\_27a)).((p\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2Eexists}\ A\_27a)\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V0x))\ (ap\ (c\_2Elbtree\_2Ebf\_\_flatten\ A\_27a)\ V1tlist))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ (ty\_2Elbtree\_2Elbtree\ A\_27a))\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V0x))\ V1tlist)))))$$