

thm_2Elbtree_2Elbtree__strong__bisimulation
(TMLmEWnvaPeFXg-
pjFMV6Ehg9LxFpgWfjkCe)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let $ty_2Elbtree_2Elbtree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elbtree_2Elbtree A0) \quad (3)$$

Let $c_2Elbtree_2Elbtree_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elbtree_2Elbtree_rep A.27a \in ((ty_2Eoption_2Eoption A.27a)^{(ty_2Elist_2Elist 2)})^{(ty_2Elbtree_2Elbtree A.27a)} \quad (4)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap$
Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 9 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 10 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b)^{(A_27b)^{(ty_2Elist_2Elist\ A_27a)^{A_27a}}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)} \quad (9)$$

Definition 11 We define $c_2Elbtree_2ENDrep$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in ((ty_2Eoption_2Eoption_ABS$

Let $c_2Elbtree_2Elbtree_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elbtree_2Elbtree_abs\ A_27a \in ((ty_2Elbtree_2Elbtree\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)}}) \quad (10)$$

Definition 12 We define $c_2Elbtree_2END$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in (ty_2Elbtree_2Elbtree_abs$

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum$

Definition 16 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eoption_2Eoption_ABS$

Definition 17 We define $c_2Elbtree_2ELfrep$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0l \in A_27a. (c_2Eoption_2ENONE$

Definition 18 We define $c_2Elbtree_2Elf$ to be $\lambda A_27a : \iota.(ap (c_2Elbtree_2Elbtree_abs A_27a) (c_2Elbtree_2Elf))$

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) (c_2Ebool_2E_3F))))$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.(ap (c_2Ebool_2E_5C_2F) (V0t1 V1t2 V2t)))))$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in A_27a. (\exists V1x \in A_27a. (V1x = V0a))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Elbtree_2Elbtree \\ A_27a).((V0t = (c_2Elbtree_2ELf\ A_27a)) \vee (\exists V1a \in A_27a. \\ (\exists V2t1 \in (ty_2Elbtree_2Elbtree\ A_27a).(\exists V3t2 \in (\\ ty_2Elbtree_2Elbtree\ A_27a).(V0t = (ap\ (ap\ (ap\ (c_2Elbtree_2END \\ A_27a)\ V1a)\ V2t1)\ V3t2))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in A_27a.(\forall V1t1 \in \\ (ty_2Elbtree_2Elbtree\ A_27a).(\forall V2t2 \in (ty_2Elbtree_2Elbtree \\ A_27a).(\neg((c_2Elbtree_2ELf\ A_27a) = (ap\ (ap\ (ap\ (c_2Elbtree_2END \\ A_27a)\ V0a)\ V1t1)\ V2t2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in A_27a.(\forall V1t1 \in \\ (ty_2Elbtree_2Elbtree\ A_27a).(\forall V2u1 \in (ty_2Elbtree_2Elbtree \\ A_27a).(\forall V3a2 \in A_27a.(\forall V4t2 \in (ty_2Elbtree_2Elbtree \\ A_27a).(\forall V5u2 \in (ty_2Elbtree_2Elbtree\ A_27a).(((ap\ (ap \\ (ap\ (c_2Elbtree_2END\ A_27a)\ V0a1)\ V1t1)\ V2u1) = (ap\ (ap\ (ap\ (c_2Elbtree_2END \\ A_27a)\ V3a2)\ V4t2)\ V5u2)) \Leftrightarrow ((V0a1 = V3a2) \wedge ((V1t1 = V4t2) \wedge (V2u1 = \\ V5u2)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Elbtree_2Elbtree \\ A_27a).(\forall V1u \in (ty_2Elbtree_2Elbtree\ A_27a).((V0t = V1u) \Leftrightarrow \\ (\exists V2R \in ((2^{(ty_2Elbtree_2Elbtree\ A_27a)})(ty_2Elbtree_2Elbtree\ A_27a)). \\ ((p\ (ap\ (ap\ V2R\ V0t)\ V1u)) \wedge (\forall V3t \in (ty_2Elbtree_2Elbtree \\ A_27a).(\forall V4u \in (ty_2Elbtree_2Elbtree\ A_27a).((p\ (ap\ (ap \\ V2R\ V3t)\ V4u)) \Rightarrow (((V3t = (c_2Elbtree_2ELf\ A_27a)) \wedge (V4u = (c_2Elbtree_2ELf \\ A_27a))) \vee (\exists V5a \in A_27a.(\exists V6t1 \in (ty_2Elbtree_2Elbtree \\ A_27a).(\exists V7u1 \in (ty_2Elbtree_2Elbtree\ A_27a).(\exists V8t2 \in \\ (ty_2Elbtree_2Elbtree\ A_27a).(\exists V9u2 \in (ty_2Elbtree_2Elbtree \\ A_27a).((p\ (ap\ (ap\ V2R\ V6t1)\ V7u1)) \wedge ((p\ (ap\ (ap\ V2R\ V8t2)\ V9u2)) \wedge \\ ((V3t = (ap\ (ap\ (ap\ (c_2Elbtree_2END\ A_27a)\ V5a)\ V6t1)\ V8t2)) \wedge (V4u = \\ (ap\ (ap\ (ap\ (c_2Elbtree_2END\ A_27a)\ V5a)\ V7u1)\ V9u2)))))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (48)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in (ty_2Elbtree_2Elbtree \\ & A_27a).(\forall V1u \in (ty_2Elbtree_2Elbtree \ A_27a).((V0t = V1u) \Leftrightarrow \\ & (\exists V2R \in ((2^{(ty_2Elbtree_2Elbtree \ A_27a)})(ty_2Elbtree_2Elbtree \ A_27a)). \\ & ((p \ (ap \ (ap \ V2R \ V0t) \ V1u)) \wedge (\forall V3t \in (ty_2Elbtree_2Elbtree \\ & A_27a).(\forall V4u \in (ty_2Elbtree_2Elbtree \ A_27a).((p \ (ap \ (ap \\ & V2R \ V3t) \ V4u)) \Rightarrow ((V3t = V4u) \vee (\exists V5a \in A_27a.(\exists V6t1 \in \\ & (ty_2Elbtree_2Elbtree \ A_27a).(\exists V7u1 \in (ty_2Elbtree_2Elbtree \\ & A_27a).(\exists V8t2 \in (ty_2Elbtree_2Elbtree \ A_27a).(\exists V9u2 \in \\ & (ty_2Elbtree_2Elbtree \ A_27a).((p \ (ap \ (ap \ V2R \ V6t1) \ V7u1)) \wedge ((p \\ & (ap \ (ap \ V2R \ V8t2) \ V9u2)) \wedge ((V3t = (ap \ (ap \ (ap \ (c_2Elbtree_2ENd \ A_27a) \\ & V5a) \ V6t1) \ V8t2)) \wedge (V4u = (ap \ (ap \ (ap \ (c_2Elbtree_2ENd \ A_27a) \ V5a) \\ & V7u1) \ V9u2))))))))))))))))))))) \end{aligned}$$