

thm_2Elbtree_2Elbtree__ue__Axiom
 (TMKVWsPriAzp-
 nueu8wVo614N8dW8en3ZDHJ)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A$. **if** $(\exists x \in A. p (ap P x))$ **then** (the $(\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define `c_2Ebool_2E_T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1t \in A_27a. V1t)) (\lambda V2t \in A_27a. V2t))$

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))$

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a) (\lambda V1x \in A_27a. V1x))$

Definition 8 We define `c_2Ebool_2E_3F_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap c_2Ebool_2E_2F_5C A_27a) (\lambda V1t \in A_27a. V1t))$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Definition 9 We define `c_2Ebool_2E_F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$.

Definition 10 We define `c_2Ebool_2E_COND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2E_21 A_27a) (V1t1 V2t2))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (6)$$

Definition 12 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b)^{(A_27b)^{(ty_2Elist_2Elist\ A_27a)^{A_27a}}})_{A_27b})^{(ty_2Elist_2Elist\ A_27a)} \quad (7)$$

Definition 13 We define $c_2Elbtree_2ENDrep$ to be $\lambda A_27a : \iota.\lambda V0a \in A_27a.\lambda V1t1 \in ((ty_2Eoption_2Eop$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2$

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Definition 17 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c$

Definition 18 We define $c_2Elbtree_2ELfrep$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0l \in A_27a.(c_2Eoption_2ENON$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 20 We define $c_2Elbtree_2Eis_lbtree$ to be $\lambda A_27a : \iota.\lambda V0t \in ((ty_2Eoption_2Eoption\ A_27a)^{(t$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Elbtree_2Epath_follow : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elbtree_2Epa\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)})_{A_27b})^{(ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27b))} \quad (9)$$

Let $ty_2Elbtree_2Elbtree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elbtree_2Elbtree\ A0) \quad (10)$$

Let $c_2Elbtree_2Elbtree_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elbtree_2Elbtree_abs\ A_27a \in \\ ((ty_2Elbtree_2Elbtree\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)}})) \quad (11)$$

Definition 21 We define $c_2Elbtree_2Elf$ to be $\lambda A_27a : \iota.(ap\ (c_2Elbtree_2Elbtree_abs\ A_27a)\ (c_2Elbtree_2Elbtree_abs\ A_27a))$

Let $c_2Elbtree_2Elbtree_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elbtree_2Elbtree_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)})^{(ty_2Elbtree_2Elbtree\ A_27a)}) \quad (12)$$

Definition 22 We define $c_2Elbtree_2ENd$ to be $\lambda A_27a : \iota.\lambda V0a \in A_27a.\lambda V1t1 \in (ty_2Elbtree_2Elbtree_abs\ A_27a)$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (13)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (14)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b)^{(A_27b^{A_27a})})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (15)$$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (16)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (17)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (18)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (19)$$

Definition 24 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Eprod\ A_27a\ A_27b)$. Assume the following.

$$True \quad (20)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p\ V0P) \wedge (\forall V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((p (ap (c_2Ebool_2E_3F_21 \ A_27a) (\lambda V1x \in A_27a.(ap \ V0P \ V1x)))) \Leftrightarrow ((\exists V2x \in A_27a.(p (ap \ V0P \ V2x))) \wedge (\forall V3x \in A_27a. (\forall V4y \in A_27a. (((p (ap \ V0P \ V3x)) \wedge (p (ap \ V0P \ V4y))) \Rightarrow (V3x = V4y))))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg (p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND \ A_27a) V1Q) V3x_27) V5y_27)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap \ V0P \ V2x)))) \Leftrightarrow (p (ap \ V0P \ V1a)))))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in (ty_2Elbtree_2Elbtree \\
& A_27a).(ap\ (c_2Elbtree_2Elbtree_abs\ A_27a)\ (ap\ (c_2Elbtree_2Elbtree_rep \\
& A_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)}). \\
& ((p\ (ap\ (c_2Elbtree_2Eis_lbtree\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Elbtree_2Elbtree_rep \\
& A_27a)\ (ap\ (c_2Elbtree_2Elbtree_abs\ A_27a)\ V1r)) = V1r)))) \\
& \tag{46}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0g \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a \\
& (ty_2Epair_2Eprod\ A_27b\ A_27b)))^{A_27b}).(\forall V1x \in A_27b. \\
& ((ap\ (ap\ (ap\ (c_2Elbtree_2Epath_follow\ A_27a\ A_27b)\ V0g)\ V1x) \\
& (c_2Elist_2ENIL\ 2)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod \\
& A_27a\ (ty_2Epair_2Eprod\ A_27b\ A_27b))\ A_27a)\ (c_2Epair_2EFST \\
& A_27a\ (ty_2Epair_2Eprod\ A_27b\ A_27b)))\ (ap\ V0g\ V1x)))))) \wedge (\forall V2g \in \\
& ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Epair_2Eprod \\
& A_27b\ A_27b)))^{A_27b}).(\forall V3x \in A_27b.(\forall V4h \in 2.(\forall V5t \in \\
& (ty_2Elist_2Elist\ 2).((ap\ (ap\ (ap\ (c_2Elbtree_2Epath_follow \\
& A_27a\ A_27b)\ V2g)\ V3x)\ (ap\ (ap\ (c_2Elist_2ECONS\ 2)\ V4h)\ V5t)) = (\\
& ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ (ty_2Epair_2Eprod\ A_27a \\
& (ty_2Epair_2Eprod\ A_27b\ A_27b))\ (ty_2Eoption_2Eoption\ A_27a)) \\
& (ap\ V2g\ V3x))\ (c_2Eoption_2ENONE\ A_27a))\ (\lambda V6v \in (ty_2Epair_2Eprod \\
& A_27a\ (ty_2Epair_2Eprod\ A_27b\ A_27b)).(ap\ (ap\ (c_2Epair_2Epair_CASE \\
& (ty_2Eoption_2Eoption\ A_27a)\ A_27a\ (ty_2Epair_2Eprod\ A_27b\ A_27b)) \\
& V6v)\ (\lambda V7a \in A_27a.(\lambda V8v2 \in (ty_2Epair_2Eprod\ A_27b\ A_27b). \\
& (ap\ (ap\ (c_2Epair_2Epair_CASE\ (ty_2Eoption_2Eoption\ A_27a) \\
& A_27b\ A_27b)\ V8v2)\ (\lambda V9y \in A_27b.(\lambda V10z \in A_27b.(ap\ (ap\ (ap \\
& (c_2Elbtree_2Epath_follow\ A_27a\ A_27b)\ V2g)\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& A_27b)\ V4h)\ V9y)\ V10z))\ V5t)))))))))) \\
& \tag{47}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b.(\forall V1f \in ((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\
& ((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ A_27b)\ (c_2Elist_2ENIL \\
& A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A_27a.(\forall V3a1 \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V4v \in A_27b.(\forall V5f \in (\\
& (A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}).((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\
& (ap\ (ap\ V5f\ V2a0)\ V3a1)))))) \\
& \tag{48}
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}). \\ & ((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A.27a).((V0l = (c_2Elist_2ENIL\ A.27a)) \vee (\exists V1h \in A.27a.(\\ & \exists V2t \in (ty_2Elist_2Elist\ A.27a).(V0l = (ap\ (ap\ (c_2Elist_2ECONS \\ & A.27a\ V1h)\ V2t)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0v \in A.27b.(\forall V1f \in (A.27b^{A.27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & A.27a\ A.27b)\ (c_2Eoption_2ENONE\ A.27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A.27a.(\forall V3v \in A.27b.(\forall V4f \in (A.27b^{A.27a}).((ap\ (ap \\ & (ap\ (c_2Eoption_2Eoption_CASE\ A.27a\ A.27b)\ (ap\ (c_2Eoption_2ESOME \\ & A.27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(((ap\ (c_2Eoption_2ESOME\ A.27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A.27a\ A.27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A.27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\ & A.27b)\ (ap\ V0f\ V1x)))) \wedge (\forall V2f \in (A.27b^{A.27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\ & A.27a\ A.27b)\ V2f)\ (c_2Eoption_2ENONE\ A.27a)) = (c_2Eoption_2ENONE \\ & A.27b)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ & A.27a).((\exists V1x \in A.27a.(V0opt = (ap\ (c_2Eoption_2ESOME\ A.27a)\ \\ & V1x))) \vee (V0opt = (c_2Eoption_2ENONE\ A.27a)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).(\exists V1q \in A.27a. \\ & (\exists V2r \in A.27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ \\ & V1q)\ V2r)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b. (\forall V1y \in A_27c. (\forall V2f \in \\ & ((A_27a^{A_27c})^{A_27b}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\ & A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (57)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q) \vee (\neg(p\ V0p)))))))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \tag{72}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Eoption_2Eoption (ty_2Epair_2Eprod A_27b \\
& (ty_2Epair_2Eprod A_27a A_27a)))^{A_27a}). (p (ap (c_2Ebool_2E_3F_21 \\
& ((ty_2Elbtree_2Elbtree A_27b)^{A_27a})) (\lambda V1g \in ((ty_2Elbtree_2Elbtree \\
& A_27b)^{A_27a}). (ap (c_2Ebool_2E_21 A_27a) (\lambda V2x \in A_27a. (ap \\
& (ap (c_2Emin_2E_3D (ty_2Elbtree_2Elbtree A_27b)) (ap V1g V2x)) \\
& (ap (ap (ap (c_2Eoption_2Eoption_CASE (ty_2Epair_2Eprod A_27b \\
& (ty_2Epair_2Eprod A_27a A_27a)) (ty_2Elbtree_2Elbtree A_27b)) \\
& (ap V0f V2x)) (c_2Elbtree_2Elf A_27b)) (\lambda V3v \in (ty_2Epair_2Eprod \\
& A_27b (ty_2Epair_2Eprod A_27a A_27a)). (ap (ap (c_2Epair_2Epair_CASE \\
& (ty_2Elbtree_2Elbtree A_27b) A_27b (ty_2Epair_2Eprod A_27a A_27a)) \\
& V3v) (\lambda V4b \in A_27b. (\lambda V5v2 \in (ty_2Epair_2Eprod A_27a A_27a). \\
& (ap (ap (c_2Epair_2Epair_CASE (ty_2Elbtree_2Elbtree A_27b) \\
& A_27a A_27a) V5v2) (\lambda V6y \in A_27a. (\lambda V7z \in A_27a. (ap (ap (ap \\
& (c_2Elbtree_2ENd A_27b) V4b) (ap V1g V6y)) (ap V1g V7z)))))))))))))
\end{aligned}$$