

thm_2Elbtree_2Emem__cases
(TMR81ZbpCLwaafPQBBgGXp9ZLvoGUeu1C4o)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2E_7E` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let `ty_2Elbtree_2Elbtree` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elbtree_2Elbtree A0) \quad (3)$$

Let `c_2Elbtree_2Elbtree_rep` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elbtree_2Elbtree_rep A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Elist_2Elist 2)})^{(ty_2Elbtree_2Elbtree A_27a)} \quad (4)$$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (a$
Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 10 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS$
Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 11 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_$
Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Definition 12 We define $c_2Elbtree_2ENdrep$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in ((ty_2Eoption_2Eop$
Let $c_2Elbtree_2Elbtree_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elbtree_2Elbtree_abs\ A_27a \in ((ty_2Elbtree_2Elbtree\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)}}) \quad (10)$$

Definition 13 We define $c_2Elbtree_2ENd$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in (ty_2Elbtree_2Elbtree$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Elbtree_2Emem$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in A_27a. (\lambda V1a1 \in (ty_2Elbtree_2Elb$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow ((p V1y) \wedge (p V3w)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow ((p V1y) \vee (p V3w)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p (ap V0P V3x))) \Rightarrow (\exists V4x \in A_27a.(p (ap V1Q V4x)))))) \quad (16)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a0 \in A_27a.(\forall V1a1 \in (ty_2Elbtree_2Elbtree A_27a).((p (ap (ap (c_2Elbtree_2Emem A_27a) V0a0) V1a1)) \Leftrightarrow ((\exists V2t1 \in (ty_2Elbtree_2Elbtree A_27a).(\exists V3t2 \in (ty_2Elbtree_2Elbtree A_27a).(V1a1 = (ap (ap (ap (c_2Elbtree_2ENd A_27a) V0a0) V2t1) V3t2)))) \vee ((\exists V4b \in A_27a.(\exists V5t1 \in (ty_2Elbtree_2Elbtree A_27a).(\exists V6t2 \in (ty_2Elbtree_2Elbtree A_27a).((V1a1 = (ap (ap (ap (c_2Elbtree_2ENd A_27a) V4b) V5t1) V6t2)) \wedge (p (ap (ap (c_2Elbtree_2Emem A_27a) V0a0) V5t1)))))) \vee ((\exists V7b \in A_27a.(\exists V8t1 \in (ty_2Elbtree_2Elbtree A_27a).(\exists V9t2 \in (ty_2Elbtree_2Elbtree A_27a).((V1a1 = (ap (ap (ap (c_2Elbtree_2ENd A_27a) V7b) V8t1) V9t2)) \wedge (p (ap (ap (c_2Elbtree_2Emem A_27a) V0a0) V9t2)))))))))) \quad (17)$$