

thm_2Elbtree_2Emem_rules (TMEvkUxJY- RoKAdBdGDnzDNstsrUECBeow5s)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Definition 4 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow_p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_21 } 2))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Let $\text{ty_2Eoption_2Eoption} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (1)$$

Let $\text{ty_2Elist_2Elist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (2)$$

Let $\text{ty_2Elbtree_2Elbtree} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elbtree_2Elbtree } A0) \quad (3)$$

Let $\text{c_2Elbtree_2Elbtree_rep} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \text{c_2Elbtree_2Elbtree_rep } A_{27a} \in \\ & ((\text{ty_2Eoption_2Eoption } A_{27a})^{(\text{ty_2Elist_2Elist } 2)})^{(\text{ty_2Elbtree_2Elbtree } A_{27a})} \end{aligned} \quad (4)$$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (a$
Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 10 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS$
Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 11 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS$
Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Definition 12 We define $c_2Elbtree_2ENDrep$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in ((ty_2Eoption_2Eoption_ABS$
Let $c_2Elbtree_2Elbtree_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elbtree_2Elbtree_abs\ A_27a \in ((ty_2Elbtree_2Elbtree\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ 2)}}) \quad (10)$$

Definition 13 We define $c_2Elbtree_2END$ to be $\lambda A_27a : \iota. \lambda V0a \in A_27a. \lambda V1t1 \in (ty_2Elbtree_2Elbtree_abs$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Elbtree_2Emem$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in A_27a. (\lambda V1a1 \in (ty_2Elbtree_2Elbtree_abs$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \wedge (p \ V2z)) \Rightarrow ((p \ V1y) \wedge (p \ V3w)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \vee (p \ V2z)) \Rightarrow ((p \ V1y) \vee (p \ V3w)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V0P \ V2x)) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p \ (ap \ V0P \ V3x))) \Rightarrow (\exists V4x \in A_27a.(p \ (ap \ V1Q \ V4x)))))) \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0a \in A_27a.(\forall V1t1 \in \\ & (ty_2Elbtree_2Elbtree \ A_27a).(\forall V2t2 \in (ty_2Elbtree_2Elbtree \\ & A_27a).(p \ (ap \ (ap \ (c_2Elbtree_2Emem \ A_27a) \ V0a) \ (ap \ (ap \ (ap \ (c_2Elbtree_2ENd \\ & A_27a) \ V0a) \ V1t1) \ V2t2)))))) \wedge ((\forall V3a \in A_27a.(\forall V4b \in \\ & A_27a.(\forall V5t1 \in (ty_2Elbtree_2Elbtree \ A_27a).(\forall V6t2 \in \\ & (ty_2Elbtree_2Elbtree \ A_27a).((p \ (ap \ (ap \ (c_2Elbtree_2Emem \ A_27a) \\ & V3a) \ V5t1)) \Rightarrow (p \ (ap \ (ap \ (c_2Elbtree_2Emem \ A_27a) \ V3a) \ (ap \ (ap \ (ap \\ & (c_2Elbtree_2ENd \ A_27a) \ V4b) \ V5t1) \ V6t2)))))) \wedge (\forall V7a \in \\ & A_27a.(\forall V8b \in A_27a.(\forall V9t1 \in (ty_2Elbtree_2Elbtree \\ & A_27a).(\forall V10t2 \in (ty_2Elbtree_2Elbtree \ A_27a).((p \ (ap \\ & (ap \ (c_2Elbtree_2Emem \ A_27a) \ V7a) \ V10t2)) \Rightarrow (p \ (ap \ (ap \ (c_2Elbtree_2Emem \\ & A_27a) \ V7a) \ (ap \ (ap \ (ap \ (c_2Elbtree_2ENd \ A_27a) \ V8b) \ V9t1) \ V10t2)))))))))) \end{aligned}$$