thm_2Elbtree_2Emem__strongind (TMZ9s9acGuxALypN2YZitFhhJC4xkMt1gSF)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (1)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (2)$$

Let $ty\_2Elbtree\_2Elbtree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elbtree\_2Elbtree\ A0) \quad (3)$$

Let $c\_2Elbtree\_2Elbtree\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})^{(ty\_2Elbtree\_2Elbtree\ A\_27a)}) \quad (4)$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{5}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{6}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{7}$$

Definition 10 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{8}$$

Definition 11 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Elist\_2Elist\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Definition 12 *We define* c_2Elbtree_2ENdrep *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in ((ty\_2Eoption\_2Eop$

Let $c\_2Elbtree\_2Elbtree\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elbtree\_2Elbtree\_\_abs\ A\_27a \in ((ty\_2Elbtree\_2Elbtree\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Elist\_2Elist\ 2)})}) \tag{10}$$

Definition 13 *We define* c_2Elbtree_2ENd *to be* $\lambda A\_27a : \iota.\lambda V0a \in A\_27a.\lambda V1t1 \in (ty\_2Elbtree\_2Elbtree$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 16 *We define* c_2Elbtree_2Emem *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in A\_27a.(\lambda V1a1 \in (ty\_2Elbtree\_2Elb$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{14}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{16}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0mem\_27 \in ((2^{(ty\_2Elbtree\_2Elbtree\ A\_27a)})^{A\_27a}).(((\forall V1a \in A\_27a.(\forall V2t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V3t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(p\ (ap\ (ap\ V0mem\_27\ V1a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V1a)\ V2t1)\ V3t2)))))) \wedge ((\forall V4a \in A\_27a.(\forall V5b \in A\_27a.(\forall V6t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V7t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V4a)\ V6t1)) \wedge (p\ (ap\ (ap\ V0mem\_27\ V4a)\ V6t1))) \Rightarrow (p\ (ap\ (ap\ V0mem\_27\ V4a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V5b)\ V6t1)\ V7t2)))))))) \wedge (\forall V8a \in A\_27a.(\forall V9b \in A\_27a.(\forall V10t1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(\forall V11t2 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V8a)\ V11t2)) \wedge (p\ (ap\ (ap\ V0mem\_27\ V8a)\ V11t2))) \Rightarrow (p\ (ap\ (ap\ V0mem\_27\ V8a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2ENd}\ A\_27a)\ V9b)\ V10t1)\ V11t2)))))))))) \Rightarrow (\forall V12a0 \in A\_27a.(\forall V13a1 \in (ty\_2Elbtree\_2Elbtree\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Elbtree\_2Emem}\ A\_27a)\ V12a0)\ V13a1)) \Rightarrow (p\ (ap\ (ap\ V0mem\_27\ V12a0)\ V13a1)))))))$$