thm_2Elebesgue_2EEXTREAL__SUP__FUN__SEQ__MONO__IMAG (TMYvqgj8E3PwBTvfn2nMocE8pzKQNMevMev)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ecombin_2ES *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ (\mathsf{c_2Ecombin_2ES}\ A_27a\ (A_27a^{A_27a})\ A_2$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27}$

Definition 8 *We define* c_2Ecombin_2Eo *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{2}$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (3)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Erealax_2Ereal__REP__CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal__REP__CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax} \quad (6)$$

Definition 13 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (\mathsf{c_2Emin_2E_40}\ (t$

Let $c_2Erealax_2Etreal__lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Eh} \quad (7)$$

Definition 14 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}$

Definition 16 *We define* c_2Ereal_2Esup *to be* $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (\mathsf{c_2Emin_2E_40}\ ty_2Ereal$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (8)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (9)$$

Let $c_2Eextreal_2Eextreal__le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal__le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (10)$$

Definition 17 *We define* c_2Eextreal_2Eextreal__sup *to be* $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap\ (ap\ (ap\ (\mathsf{c_2E}$

Definition 18 *We define* c_2Eextreal_2Eextreal__max *to be* $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Ee$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{11}$$

Let $c_2Elebesgue_2Emax__fn__seq : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elebesgue_2Emax__fn__seq\ A_27a \in (((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum})^{((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum})}) \tag{12}$$

Let $c_2Enum_2EREP__num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP__num \in (omega^{ty_2Enum_2Enum}) \tag{13}$$

Let $c_2Enum_2ESUC__REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC__REP \in (omega^{omega}) \tag{14}$$

Let $c_2Enum_2EABS__num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{omega}) \tag{15}$$

Definition 19 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS__num$

Let $c_2Enum_2EZERO__REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO__REP \in omega \tag{16}$$

Definition 20 *We define* c_2Enum_2E0 *to be* $(ap\ c_2Enum_2EABS__num\ c_2Enum_2EZERO__REP)$.

Definition 21 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 22 *We define* c_2Ebool_2EIN *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{17}$$

Definition 23 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred__set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred__set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \tag{18}$$

Definition 24 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 26 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2E}$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{22}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{27}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{29}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ \mathsf{c_2Ebool_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ \mathsf{c_2Ebool_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{32}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{33}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{34}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p\ V0P) \wedge (\forall V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{35}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{36}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee(\\ (p\ V1B)\wedge(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\wedge((p\ V0A)\vee(p\ V2C))))))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B)\wedge\\ (p\ V2C))\vee(p\ V0A))\Leftrightarrow(((p\ V1B)\vee(p\ V0A))\wedge((p\ V2C)\vee(p\ V0A))))))) \tag{39}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow\\ ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{40}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in\\ 2.((((p\ V0x)\Leftrightarrow(p\ V1x_27))\wedge((p\ V1x_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y_27))))\Rightarrow\\ (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x_27)\Rightarrow(p\ V3y_27))))))) \tag{41}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0x \in A_27a.((ap\ (\mathsf{c_2Ecombin_2EI}\\ A_27a)\ V0x) = V0x)) \tag{42}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow\forall A_27b.nonempty\ A_27b\Rightarrow(\\ \forall V0f \in (A_27b^{A_27a}).(((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo}\ A_27a\ A_27b\\ A_27b)\ (\mathsf{c_2Ecombin_2EI}\ A_27b))\ V0f) = V0f)\wedge((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo}\\ A_27a\ A_27b\ A_27a)\ V0f)\ (\mathsf{c_2Ecombin_2EI}\ A_27a)) = V0f))) \tag{43}$$

Assume the following.

$$(\forall V0x \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\\ V0x)\ V0x))) \tag{44}$$

Assume the following.

$$(\forall V0x \in ty_2Eextreal_2Eextreal.(\forall V1y \in ty_2Eextreal_2Eextreal.\\ (((p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\\ V1y)\ V0x)))\Leftrightarrow(V0x = V1y)))) \tag{45}$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Eextreal_2Eextreal}).(\forall V1x \in ty_2Eextreal_2Eextreal.\\ ((p\ (ap\ V0p\ V1x))\Rightarrow(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V1x)\ (ap\\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V0p)))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Eextreal_2Eextreal}).(\forall V1x \in ty_2Eextreal_2Eextreal.((p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ (ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V0p))\ V1x))\Leftrightarrow(\forall V2y \in ty_2Eextreal_2Eextreal.((p\ (ap\ V0p\ V2y))\Rightarrow(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V2y)\ V1x))))))) \tag{47}$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Eextreal_2Eextreal}).(\forall V1q \in (2^{ty_2Eextreal_2Eextreal}).((\forall V2x \in ty_2Eextreal_2Eextreal.((p\ (ap\ V0p\ V2x))\Rightarrow(\exists V3y \in ty_2Eextreal_2Eextreal.((p\ (ap\ V1q\ V3y))\wedge(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V2x)\ V3y))))))\Rightarrow(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ (ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V0p))\ (ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V1q)))))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty_2Eextreal_2Eextreal.(\forall V1y \in ty_2Eextreal_2Eextreal.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V1y)\ (ap\ (ap\ \mathsf{c_2Eextreal_2Eextreal__max}\ V0x)\ V1y))))) \tag{49}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0P \in (2^{ty_2Eextreal_2Eextreal}).(\forall V1P_27 \in (2^{(ty_2Eextreal_2Eextreal^{A_27a})}).(\forall V2f \in (ty_2Eextreal_2Eextreal^{(ty_2Eextreal_2Eextreal^{A_27a})}).(((\exists V3x \in ty_2Eextreal_2Eextreal.(p\ (ap\ V0P\ V3x)))\wedge((\exists V4z \in ty_2Eextreal_2Eextreal.((\neg(V4z = c_2Eextreal_2EPosInf))\wedge(\forall V5x \in ty_2Eextreal_2Eextreal.((p\ (ap\ V0P\ V5x))\Rightarrow(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V5x)\ V4z))))))\wedge(V0P = (ap\ (ap\ (\mathsf{c_2Epred__set_2EIMAGE}\ (ty_2Eextreal_2Eextreal^{A_27a})\ ty_2Eextreal_2Eextreal)\ V2f)\ V1P_27))))\Rightarrow(\exists V6g \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}).((\forall V7n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ (ty_2Eextreal_2Eextreal^{A_27a}))\ (ap\ V6g\ V7n))\ V1P_27)))\wedge((ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ (ap\ (ap\ (\mathsf{c_2Epred__set_2EIMAGE}\ ty_2Enum_2Enum\ ty_2Eextreal_2Eextreal)\ (\lambda V8n \in ty_2Enum_2Enum.(ap\ V2f\ (ap\ V6g\ V8n))))\ (\mathsf{c_2Epred__set_2EUNIV}\ ty_2Enum_2Enum)))\ =\ (ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V0P)))))))) \tag{50}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow((\forall V0g \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}).(\forall V1x \in A_27a.((ap\ (ap\ (ap\ (c_2Elebesgue_2Emax__fn__seq\ A_27a)\ V0g)\ \mathsf{c_2Enum_2E0})\ V1x) = (ap\ (ap\ V0g\ \mathsf{c_2Enum_2E0})\ V1x))))\wedge(\forall V2g \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}).(\forall V3n \in ty_2Enum_2Enum.(\forall V4x \in A_27a.((ap\ (ap\ (ap\ (c_2Elebesgue_2Emax__fn__seq\ A_27a)\ V2g)\ (ap\ \mathsf{c_2Enum_2ESUC}\ V3n))\ V4x) = (ap\ (ap\ \mathsf{c_2Eextreal_2Eextreal__max}\ (ap\ (ap\ (ap\ (c_2Elebesgue_2Emax__fn__seq\ A_27a)\ V2g)\ V3n)\ V4x))\ (ap\ (ap\ V2g\ (ap\ \mathsf{c_2Enum_2ESUC}\ V3n))\ V4x))))))) \tag{51}$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ \mathsf{c_2Enum_2E0}))\wedge (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c_2Enum_2ESUC}\ V1n)))))))\Rightarrow(\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \tag{52}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V1x)\ V0P))\Leftrightarrow(p\ (ap\ V0P\ V1x))))) \tag{53}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0x \in A_27a.(p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V0x)\ (\mathsf{c_2Epred__set_2EUNIV}\ A_27a)))) \tag{54}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow\forall A_27b.nonempty\ A_27b\Rightarrow(\forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}).((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c_2Epred__set_2EIMAGE}\ A_27a\ A_27b)\ V2f)\ V1s)))\Leftrightarrow(\exists V3x \in A_27a.((V0y = (ap\ V2f\ V3x))\wedge (p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ A_27a)\ V3x)\ V1s)))))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{70}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{(ty_2Eextreal_2Eextreal^{A_27a})}).$$
$$(\forall V1P \in (2^{ty_2Eextreal_2Eextreal}).(\forall V2P_27 \in ($$
$$2^{(ty_2Eextreal_2Eextreal^{A_27a})}).(((\exists V3x \in ty_2Eextreal_2Eextreal.$$
$$(p\ (ap\ V1P\ V3x)))\wedge((\exists V4z \in ty_2Eextreal_2Eextreal.((\neg($$
$$V4z = c_2Eextreal_2EPosInf))\wedge(\forall V5x \in ty_2Eextreal_2Eextreal.$$
$$((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ V5x)\ V4z))))))\wedge$$
$$((V1P = (ap\ (ap\ (\mathsf{c_2Epred__set_2EIMAGE}\ (ty_2Eextreal_2Eextreal^{A_27a})$$
$$ty_2Eextreal_2Eextreal)\ V0f)\ V2P_27))\wedge((\forall V6g1 \in (ty_2Eextreal_2Eextreal^{A_27a}).$$
$$(\forall V7g2 \in (ty_2Eextreal_2Eextreal^{A_27a}).(((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}$$
$$(ty_2Eextreal_2Eextreal^{A_27a}))\ V6g1)\ V2P_27))\wedge((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}$$
$$(ty_2Eextreal_2Eextreal^{A_27a}))\ V7g2)\ V2P_27))\wedge(\forall V8x \in$$
$$A_27a.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ (ap\ V6g1\ V8x))\ (ap\ V7g2$$
$$V8x))))))\Rightarrow(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ (ap\ V0f\ V6g1))$$
$$(ap\ V0f\ V7g2))))))\wedge(\forall V9g1 \in (ty_2Eextreal_2Eextreal^{A_27a}).$$
$$(\forall V10g2 \in (ty_2Eextreal_2Eextreal^{A_27a}).(((p\ (ap\ (ap\ ($$
$$\mathsf{c_2Ebool_2EIN}\ (ty_2Eextreal_2Eextreal^{A_27a}))\ V9g1)\ V2P_27))\wedge$$
$$(p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ (ty_2Eextreal_2Eextreal^{A_27a}))\ V10g2)$$
$$V2P_27)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ (ty_2Eextreal_2Eextreal^{A_27a}))$$
$$(\lambda V11x \in A_27a.(ap\ (ap\ \mathsf{c_2Eextreal_2Eextreal__max}\ (ap\ V9g1$$
$$V11x))\ (ap\ V10g2\ V11x))))\ V2P_27)))))))))\Rightarrow(\exists V12g \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}$$
$$((\forall V13n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ (\mathsf{c_2Ebool_2EIN}\ (ty_2Eextreal_2Eextreal^{A_27a}))$$
$$(ap\ V12g\ V13n))\ V2P_27)))\wedge((\forall V14x \in A_27a.(\forall V15n \in$$
$$ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal__le\ (ap\ (ap\ V12g$$
$$V15n)\ V14x))\ (ap\ (ap\ V12g\ (ap\ \mathsf{c_2Enum_2ESUC}\ V15n))\ V14x)))))\wedge(($$
$$ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ (ap\ (ap\ (\mathsf{c_2Epred__set_2EIMAGE}$$
$$ty_2Enum_2Enum\ ty_2Eextreal_2Eextreal)\ (\lambda V16n \in ty_2Enum_2Enum.$$
$$(ap\ V0f\ (ap\ V12g\ V16n))))\ (\mathsf{c_2Epred__set_2EUNIV}\ ty_2Enum_2Enum))) =$$
$$(ap\ \mathsf{c_2Eextreal_2Eextreal__sup}\ V1P)))))))))$$