

thm\_2Elebesgue\_2ERN\_lemma2  
(TMKbmMDb4gzBkDfhkSai3n47b5Dc5vT5gED)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 7** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a})).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 9** We define  $c\_2Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 10** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a})).(ap V1f V0x))$

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b}})^{A\_27a}) \quad (3)$$

**Definition 14** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (4)$$

**Definition 15** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in A\_27b.(ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ f\ s)$

**Definition 16** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ P)))$

**Definition 17** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ P)$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (5)$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealx\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealx\_2Ereal^{(2^{A\_27a})}))})}) \quad (6)$$

**Definition 18** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27a}).(ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b)\ f\ g)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealx\_2Ereal^{(ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (9)$$

**Definition 20** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 21** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 22** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num)$

**Definition 23** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2)))$

**Definition 26** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (13)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_Emin\_2E\_40) (t))$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (16)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal)$



Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})) \quad (24)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (25)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (26)$$

**Definition 38** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A-27b})^{A-27b}))})_{A\_27a})(A\_27a^{A-27b})) \quad (27)$$

**Definition 39** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 40** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets\ A\_27a \in (((2^{(2^A-27a)})^{(ty\_2Epair\_2Eprod\ (2^A-27a)\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27a)}))})) \quad (28)$$

**Definition 41** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 42** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^A-27a).\lambda V1t \in (2^A-27a).$

**Definition 43** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^A-27a).\lambda V1t \in (2^A-27a).$

**Definition 44** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^A-27a).\lambda V1Q \in ($

**Definition 45** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 46** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^A-27a)\ (ty\_2$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in (((2^A-27a)^{(ty\_2Epair\_2Eprod\ (2^A-27a)\ (ty\_2Epair\_2Eprod\ (2^{(2^A-27a)})\ (ty\_2Erealax\_2Ereal^{(2^A-27a)}))})) \quad (29)$$

**Definition 47** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$   
 Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in ( \quad (30)$$

$$(2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))})$$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (31)$$

**Definition 48** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap ($

**Definition 49** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 50** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b})$

**Definition 51** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F$

**Definition 52** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c$

**Definition 53** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 54** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 55** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2E$

**Definition 56** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 57** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E3F$

Let  $c\_2Eprim\_rec\_2ESIMP\_REC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eprim\_rec\_2ESIMP\_REC A\_27a \in ( \quad (32)$$

$$(((A\_27a^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})})^{A\_27a}))$$

**Definition 58** We define  $c\_2Eprim\_rec\_2EPRIM\_REC\_FUN$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a})$

**Definition 59** We define  $c\_2Eprim\_rec\_2EPRIM\_REC$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1f \in ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a})$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (33)$$

**Definition 60** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B$

**Definition 61** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 62** We define  $c\_2\text{Earithmetic\_2EBIT1}$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 63** We define  $c\_2\text{Earithmetic\_2ENUMERAL}$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2\text{Erealax\_2Etreax\_inv} : \iota$  be given. Assume the following.

$$c\_2\text{Erealax\_2Etreax\_inv} \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (34)$$

**Definition 64** We define  $c\_2\text{Erealax\_2Einv}$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2\text{Erealax\_2Etreax\_mul} : \iota$  be given. Assume the following.

$$c\_2\text{Erealax\_2Etreax\_mul} \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (35)$$

**Definition 65** We define  $c\_2\text{Erealax\_2Ereal\_mul}$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 66** We define  $c\_2\text{Ereal\_2E\_2F}$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2\text{Ereal\_2Epow} : \iota$  be given. Assume the following.

$$c\_2\text{Ereal\_2Epow} \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0m) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \quad (37)$$

Assume the following.

$$True \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (44)$$

Assume the following.

$$((\forall V0t \in 2.(\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (47)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27b^{A.27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A.27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (50)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (\forall V3x \in A.27a.((p \ (ap \ V0P \ V3x)) \Rightarrow (p \ (ap \ V1Q \ V3x)))))) \Rightarrow (p \ (ap \ V1Q \ (ap \ (c.2Emin_2E.40 \ A.27a) \ V0P)))))) \quad (51)$$



Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\forall V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a. ((p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (54)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (58)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c.2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (60)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\
& A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\
& A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1v \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod \\
& (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). (\forall V2e \in \\
& ty\_2Erealax\_2Ereal.(((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space \\
& A\_27a)\ V0m)) \wedge ((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space\ A\_27a)\ V1v)) \wedge \\
& ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ V2e)) \wedge ((ap\ (c\_2Emeasure\_2Em\_space\ A\_27a)\ V1v) = \\
& (ap\ (c\_2Emeasure\_2Em\_space\ A\_27a)\ V0m)) \wedge ((ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& A\_27a)\ V1v) = (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A\_27a)\ V0m)))))) \Rightarrow \\
& (\exists V3A \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})) \\
& V3A)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A\_27a)\ V0m))) \wedge ((p\ ( \\
& ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ ( \\
& ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ (ap\ (c\_2Emeasure\_2Em\_space \\
& A\_27a)\ V0m)))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V1v)\ (ap\ (c\_2Emeasure\_2Em\_space \\
& A\_27a)\ V0m))))\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure \\
& A\_27a)\ V0m)\ V3A))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V1v)\ V3A)))) \wedge \\
& (\forall V4B \in (2^{A\_27a}).(((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a})) \\
& V4B)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A\_27a)\ V0m))) \wedge ((p\ (ap \\
& (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V4B)\ V3A))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& (ap\ c\_2Erealax\_2Ereal\_neg\ V2e))\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub \\
& (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V4B))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure \\
& A\_27a)\ V1v)\ V4B)))))))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in \\
& (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E2C \\
& (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}). (\forall V1sts \in \\
& \quad (2^{(2^{A-27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A-27a})}). \\
& \quad ((ap\ (c\_2Emeasure\_2Em\_space\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ( \\
& \quad 2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad \quad V1sts)\ V2mu))) = V0sp)))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}). (\forall V1sts \in \\
& \quad (2^{(2^{A-27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A-27a})}). \\
& \quad ((ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad \quad V1sts)\ V2mu))) = V1sts)))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0sp \in (2^{A-27a}). (\forall V1sts \in \\
& \quad (2^{(2^{A-27a})}). (\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A-27a})}). \\
& \quad ((ap\ (c\_2Emeasure\_2Emeasure\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A-27a}) \\
& \quad (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad V0sp)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))) \\
& \quad \quad V1sts)\ V2mu))) = V2mu)))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0X \in (2^{A-27a}). (\forall V1P \in \\
& \quad (2^{(2^{A-27a})}). ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A.27a)\ V0X) \\
& \quad (ap\ (c\_2Epred\_set\_2EBIGINTER\ A.27a)\ V1P))) \Leftrightarrow (\forall V2Y \in (2^{A-27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a})\ V2Y)\ V1P))) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\
& \quad \quad A.27a)\ V0X)\ V2Y))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A-27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A-27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))). \\
& \quad (\forall V1s \in (2^{A-27a}). (\forall V2t \in (2^{A-27a}). (((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space \\
& \quad A.27a)\ V0m)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a})\ V1s)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& \quad A.27a)\ V0m)))) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a})\ V2t)\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets \\
& \quad A.27a)\ V0m)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a})\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& \quad \quad A.27a)\ V1s)\ V2t))\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ \\
& \quad \quad \quad V0m))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& ((p (ap (c\_2Emeasure\_2Emeasure\_space\ A.27a)\ V0m)) \Rightarrow (p (ap (ap \\
& (c\_2Ebool\_2EIN (2^{A.27a})) (ap (c\_2Emeasure\_2Em\_space\ A.27a) \\
& V0m)) (ap (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ V0m))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (2^{(2^{A.27a})})). ((p (ap (c\_2Emeasure\_2Esigma\_algebra \\
& A.27a)\ V0a)) \Rightarrow ((p (ap (ap (c\_2Emeasure\_2Esubset\_class\ A.27a) \\
& (ap (c\_2Emeasure\_2Espace\ A.27a)\ V0a)) (ap (c\_2Emeasure\_2Esubsets \\
& A.27a)\ V0a))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a})) (c\_2Epred\_set\_2EEMPTY \\
& A.27a)) (ap (c\_2Emeasure\_2Esubsets\ A.27a)\ V0a))) \wedge ((\forall V1s \in \\
& (2^{A.27a}). ((p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a})) V1s) (ap (c\_2Emeasure\_2Esubsets \\
& A.27a)\ V0a))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a})) (ap (ap (c\_2Epred\_set\_2EDIFF \\
& A.27a) (ap (c\_2Emeasure\_2Espace\ A.27a)\ V0a)) V1s) (ap (c\_2Emeasure\_2Esubsets \\
& A.27a)\ V0a)))))) \wedge (\forall V2f \in ((2^{A.27a})^{ty\_2Enum\_2Enum}). (( \\
& p (ap (ap (c\_2Ebool\_2EIN ((2^{A.27a})^{ty\_2Enum\_2Enum})) V2f) (ap ( \\
& ap (c\_2Epred\_set\_2EFUNSET\ ty\_2Enum\_2Enum (2^{A.27a})) (c\_2Epred\_set\_2EUNIV \\
& ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Esubsets\ A.27a)\ V0a)))))) \Rightarrow \\
& (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a})) (ap (c\_2Epred\_set\_2EBIGINTER \\
& A.27a) (ap (ap (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum (2^{A.27a})) \\
& V2f) (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))))) (ap (c\_2Emeasure\_2Esubsets \\
& A.27a)\ V0a)))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1s \in ((2^{A.27a})^{ty\_2Enum\_2Enum}). (((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A.27a)\ V0m)) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (p (ap (ap (c\_2Ebool\_2EIN \\
& (2^{A.27a})) (ap V1s\ V2n)) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a})) (ap (c\_2Epred\_set\_2EBIGINTER \\
& A.27a) (ap (ap (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum (2^{A.27a})) \\
& V1s) (c\_2Epred\_set\_2EUNIV\ ty\_2Enum\_2Enum)))))) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) (ty\_2Erealx\_2Ereal^{(2^{A_{.27a}})}))) \\
& (\forall V1f \in ((2^{A_{.27a}})^{ty\_2Enum\_2Enum}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A_{.27a}) V0m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ((2^{A_{.27a}})^{ty\_2Enum\_2Enum})) \\
& V1f) (ap (ap (c\_2Epred\_set\_2EFUNSET ty\_2Enum\_2Enum (2^{A_{.27a}}) \\
& (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A_{.27a}) V0m)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& A_{.27a}) (ap V1f (ap c\_2Enum\_2ESUC V2n))) (ap V1f V2n)))))) \Rightarrow (p (ap \\
& (ap c\_2Eseq\_2E\_2D\_2D\_3E (ap (ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum \\
& ty\_2Erealx\_2Ereal (2^{A_{.27a}}) (ap (c\_2Emeasure\_2Emeasure A_{.27a}) \\
& V0m)) V1f)) (ap (ap (c\_2Emeasure\_2Emeasure A_{.27a}) V0m) (ap (c\_2Epred\_set\_2EBIGINTER \\
& A_{.27a}) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum (2^{A_{.27a}}) \\
& V1f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum)))))))))) \\
& (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) (ty\_2Erealx\_2Ereal^{(2^{A_{.27a}})}))) \\
& (\forall V1s \in (2^{A_{.27a}}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A_{.27a}) V0m)) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A_{.27a}}) V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A_{.27a}) V0m)))) \Rightarrow (p (ap (c\_2Emeasure\_2Emeasure\_space A_{.27a}) ( \\
& ap (ap (c\_2Epair\_2E\_2C (2^{A_{.27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{.27a}})}) \\
& (ty\_2Erealx\_2Ereal^{(2^{A_{.27a}})}))) V1s) (ap (ap (c\_2Epair\_2E\_2C \\
& (2^{(2^{A_{.27a}})}) (ty\_2Erealx\_2Ereal^{(2^{A_{.27a}})}) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& (2^{A_{.27a}}) (2^{A_{.27a}}) (\lambda V2t \in (2^{A_{.27a}}).(ap (ap (c\_2Epred\_set\_2EINTER \\
& A_{.27a}) V1s) V2t))) (ap (c\_2Emeasure\_2Emeasurable\_sets A_{.27a}) \\
& V0m))) (ap (c\_2Emeasure\_2Emeasure A_{.27a}) V0m)))))) \\
& (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \\
& (74)
\end{aligned}$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(p (ap (ap (c\_2Ebool\_2EIN \\
A_{.27a}) V0x) (c\_2Epred\_set\_2EUNIV A_{.27a})))) \quad (75)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\
& (2^{A_{.27a}}).(\forall V2u \in (2^{A_{.27a}}).(((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& A_{.27a}) V0s) V1t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET A_{.27a}) V1t) \\
& V2u)))) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET A_{.27a}) V0s) V2u)))))) \\
& (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& (2^{A\_27a}).(p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& A\_27a)\ V0s)\ V1t))) \wedge (\forall V2s \in (2^{A\_27a}).(\forall V3t \in \\
& (2^{A\_27a}).(p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER \\
& A\_27a)\ V3t)\ V2s))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V0s)\ V1t)) \Rightarrow \\
& ((ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V1t)\ V0s) = V0s)))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). \\
& ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& A\_27a\ A\_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(\forall V1P \in (2^{A\_27a}).(\forall V2Q \in \\
& (2^{A\_27b}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (A\_27b^{A\_27a}))\ V0f)\ (ap\ (ap \\
& (c\_2Epred\_set\_2EFUNSET\ A\_27a\ A\_27b)\ V1P)\ V2Q)))) \Leftrightarrow (\forall V3x \in \\
& A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& A\_27b)\ (ap\ V0f\ V3x))\ V2Q))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1f \in \\
& ((A\_27a^{ty\_2Enum\_2Enum})^{A\_27a}).(((ap\ (ap\ (ap\ (c\_2Eprim\_rec\_2EPRIM\_REC \\
& A\_27a)\ V0x)\ V1f)\ c\_2Enum\_2E0) = V0x) \wedge (\forall V2m \in ty\_2Enum\_2Enum. \\
& ((ap\ (ap\ (ap\ (c\_2Eprim\_rec\_2EPRIM\_REC\ A\_27a)\ V0x)\ V1f)\ (ap\ c\_2Enum\_2ESUC \\
& V2m)) = (ap\ (ap\ V1f\ (ap\ (ap\ (ap\ (c\_2Eprim\_rec\_2EPRIM\_REC\ A\_27a) \\
& V0x)\ V1f)\ V2m))\ V2m))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealx\_2Ereal.(\forall V1y \in ty\_2Erealx\_2Ereal. \\
& (\neg((p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt \\
& V1y)\ V0x))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c\_2Erealx\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))
\end{aligned} \tag{83}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((ap\ c\_2Erealax\_2Ereal\_neg (ap (ap\ c\_2Ereal\_2Ereal\_sub\ V0x)\ V1y)) = (ap (ap\ c\_2Ereal\_2Ereal\_sub\ V1y)\ V0x)))) \quad (84)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap\ c\_2Erealax\_2Ereal\_lt (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) (ap\ c\_2Erealax\_2Ereal\_neg\ V1y))) \Leftrightarrow (p (ap (ap\ c\_2Erealax\_2Ereal\_lt\ V1y)\ V0x)))))) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap\ c\_2Ereal\_2Ereal\_lte (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) (ap\ c\_2Erealax\_2Ereal\_neg\ V1y))) \Leftrightarrow (p (ap (ap\ c\_2Ereal\_2Ereal\_lte\ V1y)\ V0x)))))) \quad (86)$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. ((p (ap (ap\ c\_2Erealax\_2Ereal\_lt\ V1x)\ V0y)) \Leftrightarrow (\neg (p (ap (ap\ c\_2Ereal\_2Ereal\_lte\ V0y)\ V1x)))))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ c\_2Erealax\_2Ereal\_neg (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) = V0x)) \quad (88)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (89)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (90)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (91)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (92)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (93)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg( \\
& p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee \\
& ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee \\
& (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge (( \\
& \neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{98}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{99}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \tag{100}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow (\neg(p \vee 0p)))) \tag{101}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \tag{102}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p))) \tag{103}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1x0 \in \\
& ty\_2Erealax\_2Ereal.(\forall V2y \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (\forall V3y0 \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Eseq\_2E\_2D\_2D\_3E \\
& V0x) V1x0)) \wedge (p (ap (ap c\_2Eseq\_2E\_2D\_2D\_3E V2y) V3y0))) \Rightarrow (p (ap \\
& (ap c\_2Eseq\_2E\_2D\_2D\_3E (\lambda V4n \in ty\_2Enum\_2Enum.(ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap V0x V4n)) (ap V2y V4n)))) (ap (ap c\_2Ereal\_2Ereal\_sub V1x0) \\
& V3y0))))))))) \\
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1x \in \\
& ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.(((\forall V3n \in \\
& ty\_2Enum\_2Enum.(p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V0f (ap (ap \\
& c\_2Earithmetic\_2E\_2B V3n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) (ap V0f V3n)))) \wedge (p (ap (ap c\_2Eseq\_2E\_2D\_2D\_3E \\
& V0f) V1x))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V1x) (ap V0f V2n))))))))) \\
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Epow \\
& (ap (ap c\_2Ereal\_2E\_2F (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \\
& V0n)))) \\
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0e \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0e)) \Rightarrow (\exists V1n \in \\
& ty\_2Enum\_2Enum.(p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Ereal\_2Epow \\
& (ap (ap c\_2Ereal\_2E\_2F (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \\
& V1n)) V0e)))) \\
\end{aligned} \tag{107}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty\_2Epair\_2Eprod} \\
& (2^{A_{27a}}) (\text{ty\_2Epair\_2Eprod} (2^{(2^{A_{27a}})}) (\text{ty\_2Erealax\_2Ereal}^{(2^{A_{27a}})}))))). \\
& (\forall V1v \in (\text{ty\_2Epair\_2Eprod} (2^{A_{27a}}) (\text{ty\_2Epair\_2Eprod} \\
& (2^{(2^{A_{27a}})}) (\text{ty\_2Erealax\_2Ereal}^{(2^{A_{27a}})}))))). (((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A_{27a} V0m)) \wedge (p (ap (c\_2Emeasure\_2Emeasure\_space A_{27a} V1v)) \wedge \\
& (((ap (c\_2Emeasure\_2Em\_space A_{27a} V1v) = (ap (c\_2Emeasure\_2Em\_space \\
& A_{27a} V0m)) \wedge (ap (c\_2Emeasure\_2Emeasurable\_sets A_{27a} V1v) = \\
& (ap (c\_2Emeasure\_2Emeasurable\_sets A_{27a} V0m)))))) \Rightarrow (\exists V2A \in \\
& (2^{A_{27a}}). ((p (ap (ap (c\_2Ebool\_2EIN (2^{A_{27a}}) V2A) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A_{27a} V0m)))) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap (ap (c\_2Emeasure\_2Emeasure A_{27a} V0m) (ap (c\_2Emeasure\_2Em\_space \\
& A_{27a} V0m)))) (ap (ap (c\_2Emeasure\_2Emeasure A_{27a} V1v) (ap (c\_2Emeasure\_2Em\_space \\
& A_{27a} V0m)))) (ap (ap c\_2Ereal\_2Ereal\_sub (ap (ap (c\_2Emeasure\_2Emeasure \\
& A_{27a} V0m) V2A)) (ap (ap (c\_2Emeasure\_2Emeasure A_{27a} V1v) V2A)))))) \wedge \\
& (\forall V3B \in (2^{A_{27a}}). ((p (ap (ap (c\_2Ebool\_2EIN (2^{A_{27a}}) \\
& V3B) (ap (c\_2Emeasure\_2Emeasurable\_sets A_{27a} V0m))) \wedge (p (ap \\
& (ap (c\_2Epred\_set\_2ESUBSET A_{27a} V3B) V2A))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap (ap (c\_2Emeasure\_2Emeasure A_{27a} V0m) V3B)) (ap (ap (c\_2Emeasure\_2Emeasure \\
& A_{27a} V1v) V3B)))))))))))))
\end{aligned}$$