

thm\_2Elebesgue\_2Integrable\_\_fn\_\_plus  
(TMQAT6nrJAtLyZmdZT8QPhS5nDF6qGm14cp)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{2}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{3}$$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{4}$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{5}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in & ((2^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \end{aligned} \quad (7)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in \\ & ((2^{A\_27a})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \end{aligned} \quad (8)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (9)$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 9** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (13)$$

**Definition 10** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (14)$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (15)$$

**Definition 11** We define  $c\_2Eextreal\_2Eextreal\_lt$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then**  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap V1t1 V2t2) P)))$

**Definition 14** We define  $c\_2Emeasure\_2Efn\_minus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a})$

**Definition 15** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x) P))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (16)$$

**Definition 16** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b}).(ap V1s (c\_2Epred\_set\_2EIMAGE f))$

**Definition 17** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (c\_2Emin\_2E\_40 P)))$

**Definition 18** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap V0P (c\_2Epred\_set\_2EBIGUNION P))$

**Definition 19** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 20** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap V1t (c\_2Ebool\_2E\_3F s))$

**Definition 21** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap V1t (c\_2Ebool\_2E\_3F s))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (17)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (18)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap V0a (c\_2Emin\_2E\_40 (c\_2Erealax\_2Ereal\_REP a)))$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.(ap V1T2 (c\_2Erealax\_2Ereal\_lt T1 T2))$

**Definition 24** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.(ap V1y (c\_2Ereal\_2Ereal\_lte x y))$

**Definition 25** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap V1t2 (c\_2Ebool\_2E\_21 2) (ap V2t \in 2.(ap V2t V1t2) P))))$

**Definition 26** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap V1s (c\_2Emin\_2E\_40 (c\_2Ebool\_2E\_5C\_2F s)))$

**Definition 27** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E21 (2$

**Definition 28** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (21)$$

**Definition 29** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (22)$$

**Definition 30** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 31** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 32** We define  $c\_2Emeasure\_2Eindicator\_fn$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(\lambda V1x \in A\_27a.(ap$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal} \quad (23)$$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal} \quad (24)$$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (25)$$

**Definition 33** We define  $c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2E$

**Definition 34** We define  $c\_2Emeasure\_2Epos\_simple\_fn$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (26)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (27)$$

**Definition 35** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

**Definition 36** We define  $c\_2Elebesgue\_2Epsfs$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2E$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasure A\_27a \in ( (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) (28)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (29)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (30)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} (31)$$

**Definition 37** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal$

**Definition 38** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Erealadd : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Erealadd \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (32)$$

**Definition 39** We define  $c\_2Erealax\_2Erealadd$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 40** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal$

**Definition 41** We define  $c\_2Elebesgue\_2Epos\_simple\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 42** We define  $c\_2Elebesgue\_2Epsfis$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2E$

**Definition 43** We define  $c\_2Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Emin\_2E.40 ty\_2Ereal$

**Definition 44** We define  $c\_2Eextreal\_2Eextreal\_sup$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap (ap (ap (c\_2E$

**Definition 45** We define  $c\_2Elebesgue\_2Epos\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{$

**Definition 46** We define  $c\_2Emeasure\_2Efn\_plus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).$

**Definition 47** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ( (2^{(2^{A-27a})})^{(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})}) ) \quad (33)$$

**Definition 48** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

**Definition 49** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

**Definition 50** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

**Definition 51** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ( (2^{A-27a})^{(ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})}) ) \quad (34)$$

**Definition 52** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

**Definition 53** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1sts \in (2^{(2^{A-27a})}). (ap\ (c\_2Ebool\_2E3F$

**Definition 54** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c\_2Ebool\_2E3F$

**Definition 55** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c\_2Ebool\_2E3F$

**Definition 56** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c\_2Epred\_set\_2Ebool\_2E3F$

**Definition 57** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A-27a}). \lambda V1st \in (2^{(2^{A-27a})}). (ap\ (c\_2Ebool\_2E3F$

**Definition 58** We define  $c\_2Emeasure\_2EBorel$  to be  $(ap\ (ap\ (c\_2Emeasure\_2Esigma\ ty\_2Eextreal\_2Eextreal)))$

**Definition 59** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1g \in (A\_27a^{A-27b}). (ap\ (c\_2Ebool\_2E3F$

**Definition 60** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A-27a}). \lambda V1Q \in (2^{A-27b}). (ap\ (c\_2Ebool\_2E3F$

**Definition 61** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c\_2Ebool\_2E3F$

**Definition 62** We define  $c\_2ELebesgue\_2Eintegrable$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (2^{(2^{A-27a})})). (ap\ (c\_2Ebool\_2E3F$

**Definition 63** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A-27c}). \lambda V1g \in (A\_27c^{A-27a}). (ap\ (c\_2Ebool\_2E3F$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ( (ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum)} ) \quad (35)$$

**Definition 64** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum. \lambda V1n \in ty\_2Eenum\_2Eenum. (ap\ (c\_2Ebool\_2E3F$

**Definition 65** We define  $c\_2Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 66** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (36)$$

**Definition 67** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 68** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 69** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECON$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (37)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) \end{aligned} \quad (38)$$

**Definition 70** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)) \quad (39)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (40)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in \\ ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \end{aligned} \quad (41)$$

**Definition 71** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Eetends \\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b})})))_{A\_27a}\ (A\_27a^{A\_27b}) \end{aligned} \quad (42)$$

**Definition 72** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 73** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 74** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 75** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2$

**Definition 76** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a})$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (48)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (51)$$



Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (52)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (53)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((\neg((ap\ c\_2Eextreal\_2Eextreal\_of\_num\ V0n) = c\_2Eextreal\_2ENegInf)) \wedge (\neg((ap\ c\_2Eextreal\_2Eextreal\_of\_num\ V0n) = c\_2Eextreal\_2EPosInf)))) \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\ & (2^{A_{27a}}) (ty\_2Epair\_2Eprod (2^{(2^{A_{27a}})}) (ty\_2Erealx\_2Ereal^{(2^{A_{27a}})}))) \\ & ((p (ap (c\_2Emeasure\_2Emeasure\_space\ A_{27a})\ V0m)) \Rightarrow ((ap (ap ( \\ & c\_2Elebesgue\_2Epos\_fn\_integral\ A_{27a})\ V0m) (\lambda V1x \in A_{27a}. \\ & (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0))) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num \\ & c\_2Enum\_2E0)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A_{27a}}). \\ & (\forall V1x \in A_{27a}. (p (ap (ap\ c\_2Eextreal\_2Eextreal\_le (ap\ c\_2Eextreal\_2Eextreal\_of\_num \\ & c\_2Enum\_2E0)) (ap\ V0g\ V1x)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A_{27a}}). \\ & ((\forall V1x \in A_{27a}. (p (ap (ap\ c\_2Eextreal\_2Eextreal\_le (ap \\ & c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) (ap\ V0g\ V1x)))) \Rightarrow \\ & ((ap (c\_2Emeasure\_2Efn\_plus\ A_{27a})\ V0g) = V0g))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A_{27a}}). \\ & ((\forall V1x \in A_{27a}. (p (ap (ap\ c\_2Eextreal\_2Eextreal\_le (ap \\ & c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) (ap\ V0g\ V1x)))) \Rightarrow \\ & ((ap (c\_2Emeasure\_2Efn\_minus\ A_{27a})\ V0g) = (\lambda V2x \in A_{27a}. ( \\ & ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (\text{ty\_2Epair\_2Eprod} \\
& \quad (2^{A_{27a}}) (2^{(2^{A_{27a}})})) . (\forall V1f \in (\text{ty\_2Eextreal\_2Eextreal}^{A_{27a}}) . \\
& \quad ((p (ap (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal}^{A_{27a}})) V1f) \\
& \quad (ap (ap (c\_2Emeasure\_2E measurable A_{27a} ty\_2Eextreal\_2Eextreal) \\
& \quad V0a) c\_2Emeasure\_2EBorel))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal}^{A_{27a}})) \\
& \quad (ap (c\_2Emeasure\_2Efn\_plus A_{27a} V1f)) (ap (ap (c\_2Emeasure\_2E measurable \\
& \quad A_{27a} ty\_2Eextreal\_2Eextreal) V0a) c\_2Emeasure\_2EBorel)))))) \\
& \hspace{15em} (59)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty\_2Epair\_2Eprod} \\
& \quad (2^{A_{27a}}) (\text{ty\_2Epair\_2Eprod} (2^{(2^{A_{27a}})})) (\text{ty\_2Erealax\_2Ereal}^{(2^{A_{27a}})}))) . \\
& (\forall V1f \in (\text{ty\_2Eextreal\_2Eextreal}^{A_{27a}}) . (((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& \quad A_{27a} V0m)) \wedge (p (ap (ap (c\_2ELebesgue\_2Eintegrable A_{27a} V0m) \\
& \quad V1f)))) \Rightarrow (p (ap (ap (c\_2ELebesgue\_2Eintegrable A_{27a} V0m) (ap ( \\
& \quad c\_2Emeasure\_2Efn\_plus A_{27a} V1f))))))
\end{aligned}$$