

thm_2Elebesgue_2Eintegrable__plus__minus (TMMB2qEj9DoycU6GUUXHo4fZd2vSoa3W2kh)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Let `ty_2Eextreal_2Eextreal` : ι be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let `c_2Eextreal_2ENegInf` : ι be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \tag{2}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{3}$$

Let `c_2Eextreal_2EPosInf` : ι be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \tag{4}$$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{5}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{6}$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in & ((2^{(2^{A_27a})}) (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (7)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in \\ ((2^{A_27a}) & (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (8)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in & ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (13)$$

Definition 10 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal$

Let $c_2Eextreal_2Eextreal_ainv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_ainv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \quad (14)$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (15)$$

Definition 11 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E\ V0t) c_Ebool_2E_7E))$.

Definition 13 We define $c_Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$.

Definition 14 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_Ebool_2E_21\ 2) (V1t1\ V2t2))))))$.

Definition 15 We define $c_Emeasure_2Efn_minus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a})$.

Definition 16 We define c_Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (16)$$

Definition 17 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27a)$.

Definition 18 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EIMAGE))$.

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 20 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EBIGUNION))$.

Definition 21 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EEMPTY))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (17)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (18)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_Emin_2E_40))$.

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 23 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 24 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 25 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.(ap (c_Ebool_2E_21\ 2) (V1t2\ V2t))))))$.

Definition 26 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Epred_set_2EBIGUNION))$.

Definition 27 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2$

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 29 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (22)$$

Definition 30 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 31 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 32 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal)^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (23)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal)^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (24)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (25)$$

Definition 33 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal$

Definition 34 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (26)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (27)$$

Definition 35 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 36 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}) (28)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (29)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (30)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} (31)$$

Definition 37 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal$

Definition 38 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (32)$$

Definition 39 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 40 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Definition 41 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 42 We define $c_2Elebesgue_2Epsfis$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 43 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40 ty_2Ereal$

Definition 44 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 45 We define $c_2Elebesgue_2Epos_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{$

Definition 46 We define $c_2Emeasure_2Efn_plus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a}).$

Definition 47 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (33)$$

Definition 48 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 49 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 50 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 51 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (34)$$

Definition 52 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 53 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$

Definition 54 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 55 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 56 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2Ebool_2E3F$

Definition 57 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1st \in (2^{(2^{A_27a})}). (ap\ (c_2Ebool_2E3F$

Definition 58 We define $c_2Emeasure_2EBorel$ to be $(ap\ (ap\ (c_2Emeasure_2Esigma\ ty_2Eextreal_2Eextreal_2Ebool_2E3F$

Definition 59 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 60 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1Q \in (2^{A_27a})$

Definition 61 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 62 We define $c_2ELebesgue_2Eintegrable$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 63 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a})$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum)}) \quad (35)$$

Definition 64 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum$

Definition 65 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 66 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (36)$$

Definition 67 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 68 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 69 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECON$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (37)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ A_27a\ A_27a)}) \end{aligned} \quad (38)$$

Definition 70 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \quad (39)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (40)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \\ ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \end{aligned} \quad (41)$$

Definition 71 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eetends \\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b})})))_{A_27a}\ (A_27a^{A_27b}) \end{aligned} \quad (42)$$

Definition 72 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 73 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 74 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 75 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Definition 76 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (48)$$

Assume the following.

$$((\forall V0t \in 2.((\neg \neg (p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (52)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (55)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.((\neg((ap c.2Eextreal.2Eextreal_of_num V0n) = c.2Eextreal.2ENegInf)) \wedge (\neg((ap c.2Eextreal.2Eextreal_of_num V0n) = c.2Eextreal.2EPosInf)))) \quad (56)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in (ty.2Epair.2Eprod (2^{A.27a}) (ty.2Epair.2Eprod (2^{(2^{A.27a})}) (ty.2Erealax.2Ereal^{(2^{A.27a})})))) \cdot ((p (ap (c.2Emeasure.2Emeasure_space A.27a) V0m)) \Rightarrow ((ap (ap (c.2Elebesgue.2Epos_fn_integral A.27a) V0m) (\lambda V1x \in A.27a. (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0))) = (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)))) \quad (57)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0g \in (ty.2Eextreal.2Eextreal^{A.27a}). (\forall V1x \in A.27a.(p (ap (ap c.2Eextreal.2Eextreal_le (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)) (ap (ap (c.2Emeasure.2Efn_plus A.27a) V0g) V1x)))))) \quad (58)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0g \in (ty.2Eextreal.2Eextreal^{A.27a}). (\forall V1x \in A.27a.(p (ap (ap c.2Eextreal.2Eextreal_le (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)) (ap (ap (c.2Emeasure.2Efn_minus A.27a) V0g) V1x)))))) \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ ((\forall V1x \in A.27a.(p\ (ap\ (ap\ c.2Eextreal_2Eextreal_le\ (ap \\ c.2Eextreal_2Eextreal_of_num\ c.2Enum.2E0))\ (ap\ V0g\ V1x)))) \Rightarrow \\ ((ap\ (c.2Emeasure.2Efn_plus\ A.27a)\ V0g) = V0g))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ ((\forall V1x \in A.27a.(p\ (ap\ (ap\ c.2Eextreal_2Eextreal_le\ (ap \\ c.2Eextreal_2Eextreal_of_num\ c.2Enum.2E0))\ (ap\ V0g\ V1x)))) \Rightarrow \\ ((ap\ (c.2Emeasure.2Efn_minus\ A.27a)\ V0g) = (\lambda V2x \in A.27a.(\\ ap\ c.2Eextreal_2Eextreal_of_num\ c.2Enum.2E0)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V1f) \\ (ap\ (ap\ (c.2Emeasure.2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\ V0a)\ c.2Emeasure.2EBorel))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})) \\ (ap\ (c.2Emeasure.2Efn_plus\ A.27a)\ V1f))\ (ap\ (ap\ (c.2Emeasure.2Emeasurable \\ A.27a\ ty_2Eextreal_2Eextreal)\ V0a)\ c.2Emeasure.2EBorel)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Epair_2Eprod \\ (2^{A.27a})\ (2^{(2^{A.27a})})).(\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\ ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}))\ V1f) \\ (ap\ (ap\ (c.2Emeasure.2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\ V0a)\ c.2Emeasure.2EBorel))) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})) \\ (ap\ (c.2Emeasure.2Efn_minus\ A.27a)\ V1f))\ (ap\ (ap\ (c.2Emeasure.2Emeasurable \\ A.27a\ ty_2Eextreal_2Eextreal)\ V0a)\ c.2Emeasure.2EBorel)))))) \end{aligned} \quad (63)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (65)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (74)$$

Theorem 1

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod (2^{A-27a}) (ty_2Erealax_2Ereal^{(2^{A-27a})}))).$$

$$(\forall V1f \in (ty_2Eextreal_2Eextreal^{A-27a}).((p (ap (c_2Emeasure_2Emeasure_space A.27a) V0m)) \Rightarrow ((p (ap (ap (c_2Elebesgue_2Eintegrable A.27a) V0m) V1f)) \Leftrightarrow ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A-27a}) V1f) (ap (ap (c_2Emeasure_2Emeasurable A.27a) ty_2Eextreal_2Eextreal) (ap (ap (c_2Epair_2E_2C (2^{A-27a}) (2^{(2^{A-27a})})) (ap (c_2Emeasure_2Em_space A.27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets A.27a) V0m))) c_2Emeasure_2EBorel))) \wedge ((p (ap (ap (c_2Elebesgue_2Eintegrable A.27a) V0m) (ap (c_2Emeasure_2Efn_plus A.27a) V1f))) \wedge (p (ap (ap (c_2Elebesgue_2Eintegrable A.27a) V0m) (ap (c_2Emeasure_2Efn_minus A.27a) V1f))))))))))$$