

thm_2Elebesgue_2Integrable__pos
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \tag{2}$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{3}$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \tag{4}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) \tag{5}$$

Definition 8 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b}})^{A_27a}) \quad (7)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (8)$$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 12 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (2^{(2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})})}) \quad (9)$$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p (ap P x))$
of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 15 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E$

Definition 16 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 17 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}$

Definition 18 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_3F$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (10)$$

Definition 21 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EDIFF\ s\ t))$

Definition 22 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 23 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 24 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 25 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 26 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EBIGINTER\ P))$

Definition 27 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1st \in (2^{(2^{A_27a})}).(ap\ (c_2Esigma\ s\ t))$

Definition 28 We define $c_2Emeasure_2EBorel$ to be $(ap\ (ap\ (c_2Emeasure_2Esigma\ ty_2Eextreal_2Eextreal)))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (11)$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (12)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (13)$$

Definition 29 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1g \in (A_27a^{A_27b}).(ap\ (c_2Epred_set_2EPREIMAGE\ f\ g))$

Definition 30 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Einter\ s\ t))$

Definition 31 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap\ (c_2Efunset\ P\ Q))$

Definition 32 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})).(ap\ (c_2Emeasure_2Emeasurable\ a\ b))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 33 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (17)$$

Definition 34 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_of_num)$.

Let $c_2Eextreal_2Eextreal_ainv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_ainv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \quad (18)$$

Definition 35 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ c_2Ebool_2ECOND_t1\ t2))))$.

Definition 36 We define $c_2Emeasure_2Efn_minus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a})$.

Definition 37 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ c_2Epred_set_2EDISJOINT_t1\ t2)$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (19)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (20)$$

Definition 38 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ a))$.

Let $c_2Erealax_2Etrealm : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (21)$$

Definition 39 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_lt_T1\ T2)$.

Definition 40 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(ap\ c_2Ereal_2Ereal_lte_x\ y)$.

Definition 41 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_2EINSERT_x\ s))$.

Definition 42 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ s))$.

Definition 43 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (23)$$

Definition 44 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 45 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 46 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 47 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (\lambda V1x \in A_27a. (ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (25)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (26)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (27)$$

Definition 48 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Eextreal_2Eextreal$

Definition 49 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (28)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (29)$$

Definition 50 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 51 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in (\\ & (ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}) \\ & \end{aligned} \tag{30}$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & \end{aligned} \tag{31}$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & \end{aligned} \tag{32}$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \\ & \end{aligned} \tag{33}$$

Definition 52 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 53 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ & \end{aligned} \tag{34}$$

Definition 54 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 55 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2E$

Definition 56 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2E$

Definition 57 We define $c_2Elebesgue_2Epsfis$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 58 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40 ty_2Ereal$

Definition 59 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 60 We define $c_2Elebesgue_2Epos_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{$

Definition 61 We define $c_2Emeasure_2Efn_plus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a}).$

Definition 62 We define $c_2E\text{lebesgue_2Eintegrable}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A_27a}))$

Definition 63 We define $c_2E\text{combin_2Eo}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $c_2E\text{real_2Esum} : \iota$ be given. Assume the following.

$$c_2E\text{real_2Esum} \in ((ty_2E\text{realax_2Ereal}^{(ty_2E\text{realax_2Ereal}^{ty_2E\text{enum_2Eenum}})})_{(ty_2E\text{pair_2Eprod } ty_2E\text{enum_2Eenum})}) \quad (35)$$

Definition 64 We define $c_2E\text{prim_rec_2E_3C}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Definition 65 We define $c_2E\text{arithmic_2E_3E}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Definition 66 We define $c_2E\text{arithmic_2E_3E_3D}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Let $c_2E\text{realax_2Etreall_neg} : \iota$ be given. Assume the following.

$$c_2E\text{realax_2Etreall_neg} \in ((ty_2E\text{pair_2Eprod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})_{(ty_2E\text{pair_2Eprod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})}) \quad (36)$$

Definition 67 We define $c_2E\text{realax_2Ereal_neg}$ to be $\lambda V0T1 \in ty_2E\text{realax_2Ereal}.(ap \ c_2E\text{realax_2Ereal}$

Definition 68 We define $c_2E\text{real_2Ereal_sub}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.\lambda V1y \in ty_2E\text{realax_2Ereal}$

Definition 69 We define $c_2E\text{real_2Eabs}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.(ap \ (ap \ (ap \ (c_2E\text{bool_2ECONJ}$

Let $ty_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2E\text{metric_2Emetric } A0) \quad (37)$$

Let $c_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{metric_2Emetric } A_27a \in ((ty_2E\text{metric_2Emetric } A_27a)_{(ty_2E\text{realax_2Ereal}^{(ty_2E\text{pair_2Eprod } A_27a \ A_27a)})}) \quad (38)$$

Definition 70 We define $c_2E\text{metric_2Emr1}$ to be $(ap \ (c_2E\text{metric_2Emetric } ty_2E\text{realax_2Ereal}) \ (ap \ (c_2E\text{metric_2Emetric}$

Let $c_2E\text{metric_2Edist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{metric_2Edist } A_27a \in ((ty_2E\text{realax_2Ereal}^{(ty_2E\text{pair_2Eprod } A_27a \ A_27a)}) \quad (39)$$

Let $ty_2E\text{topology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2E\text{topology_2Etopology } A0) \quad (40)$$

Let $c_2E\text{topology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{topology_2Etopology } A_27a \in ((ty_2E\text{topology_2Etopology } A_27a)^{(2^{(2^{A_27a})})}) \quad (41)$$

Definition 71 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \end{aligned} \quad (42)$$

Definition 72 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 73 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 74 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 75 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Definition 76 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a})$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & A_27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (49)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (50)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (53)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \quad (54)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg((ap\ c.2Eextreal.2Eextreal_of_num\ V0n) = c.2Eextreal.2ENegInf)) \wedge (\neg((ap\ c.2Eextreal.2Eextreal_of_num\ V0n) = c.2Eextreal.2EPosInf)))) \quad (55)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty.2Epair.2Eprod\ (2^{A.27a})\ (ty.2Epair.2Eprod\ (2^{(2^{A.27a})})\ (ty.2Erealx.2Ereal^{(2^{A.27a})}))). ((p\ (ap\ (c.2Emeasure.2Emeasure_space\ A.27a)\ V0m)) \Rightarrow ((ap\ (ap\ (c.2Elebesgue.2Epos_fn_integral\ A.27a)\ V0m)\ (\lambda V1x \in A.27a. (ap\ c.2Eextreal.2Eextreal_of_num\ c.2Enum.2E0)))) = (ap\ c.2Eextreal.2Eextreal_of_num\ c.2Enum.2E0)))) \quad (56)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty.2Eextreal.2Eextreal^{A.27a}). ((\forall V1x \in A.27a.(p\ (ap\ (ap\ c.2Eextreal.2Eextreal_le\ (ap\ c.2Eextreal.2Eextreal_of_num\ c.2Enum.2E0))\ (ap\ V0g\ V1x)))) \Rightarrow ((ap\ (c.2Emeasure.2Efn_plus\ A.27a)\ V0g) = V0g))) \quad (57)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). \\
& \quad ((\forall V1x \in A_{.27a}.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap \\
& \quad c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ V0g\ V1x)))) \Rightarrow \quad (58) \\
& \quad ((ap\ (c_2Emeasure_2Efn_minus\ A_{.27a})\ V0g) = (\lambda V2x \in A_{.27a}.(\\
& \quad ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))))))
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A_{.27a}})\ (ty_2Epair_2Eprod\ (2^{(2^{A_{.27a}})})\ (ty_2Erealax_2Ereal^{(2^{A_{.27a}})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}).(((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A_{.27a})\ V0m)) \wedge (\forall V2x \in A_{.27a}.(p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ V1f\ V2x)))))) \Rightarrow \\
& \quad ((p\ (ap\ (ap\ (c_2Elebesgue_2Eintegrable\ A_{.27a})\ V0m)\ V1f)) \Leftrightarrow ((p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A_{.27a}}))\ V1f)\ (ap \\
& \quad (ap\ (c_2Emeasure_2Emeasurable\ A_{.27a}\ ty_2Eextreal_2Eextreal) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_{.27a}})\ (2^{(2^{A_{.27a}})})))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad A_{.27a})\ V0m))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A_{.27a})\ V0m))) \\
& \quad c_2Emeasure_2EBorel))) \wedge (\neg((ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A_{.27a})\ V0m)\ V1f) = c_2Eextreal_2EPosInf))))))
\end{aligned}$$