

thm_2Elebesgue_2Eintegrable__sub (TMPqVc5eNn8fq3BzPcyFY1ah84dCzxaP5Tx)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Definition 4 We define `c_2Ebool_2E_T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))))$

Definition 6 We define `c_2Ecombin_2E_o` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27b}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V2x \in 2.V2x)) (\lambda V3x \in 2.V3x))$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Erealax_2Ereal \tag{1}$$

Let `ty_2Eenum_2Eenum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Eenum_2Eenum \tag{2}$$

Let `c_2Ereal_2Ereal_of_num` : ι be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \tag{3}$$

Let `ty_2Ehreal_2Ehreal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Ehreal_2Ehreal \tag{4}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod A0 A1) \tag{5}$$

Let `c_2Erealax_2Ereal_REP_CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{6}$$

Definition 7 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (ty_2Erealax_2Ereal_neg) a))$.
Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (7)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (9)$$

Definition 8 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$.

Definition 9 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_neg)$.

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (10)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (11)$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (12)$$

Let $c_2Eextreal_2Eextreal_sub : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_sub \in ((ty_2Eextreal_2Eextreal)^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (13)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (15)$$

Definition 10 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (17)$$

Definition 12 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Let $c_Earithmic_E_B : \iota$ be given. Assume the following.

$$c_Earithmic_E_B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (18)$$

Definition 13 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmic$

Definition 14 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Eextreal_ENormal : \iota$ be given. Assume the following.

$$c_Eextreal_ENormal \in (ty_Eextreal_Eextreal^{ty_Erealx_Ereal}) \quad (19)$$

Definition 15 We define $c_Eextreal_Eextreal_of_num$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ c_Eextreal$

Let $c_Eextreal_Eextreal_ainv : \iota$ be given. Assume the following.

$$c_Eextreal_Eextreal_ainv \in (ty_Eextreal_Eextreal^{ty_Eextreal_Eextreal}) \quad (20)$$

Let $c_Eextreal_Eextreal_add : \iota$ be given. Assume the following.

$$c_Eextreal_Eextreal_add \in ((ty_Eextreal_Eextreal^{ty_Eextreal_Eextreal})^{ty_Eextreal_Eextreal}) \quad (21)$$

Let $c_Eextreal_Eextreal_mul : \iota$ be given. Assume the following.

$$c_Eextreal_Eextreal_mul \in ((ty_Eextreal_Eextreal^{ty_Eextreal_Eextreal})^{ty_Eextreal_Eextreal}) \quad (22)$$

Let $c_Eextreal_Eextreal_le : \iota$ be given. Assume the following.

$$c_Eextreal_Eextreal_le \in ((2^{ty_Eextreal_Eextreal})^{ty_Eextreal_Eextreal}) \quad (23)$$

Definition 16 We define c_Ebool_EF to be $(ap\ (c_Ebool_E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_Emin_E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 18 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_E_3D_3D_3E\ V0t)\ c_Ebool_E_21\ 2))$

Definition 19 We define $c_Eextreal_Eextreal_lt$ to be $\lambda V0x \in ty_Eextreal_Eextreal.\lambda V1y \in ty_Eextreal$

Definition 20 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in 2.V0t2))$

Definition 21 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 22 We define $c_Emeasure_Efn_minus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_Eextreal_Eextreal^{A_27a})$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{2^{A_27a}})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (24)$$

Definition 23 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (25)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (26)$$

Definition 25 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in A_27b. (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ (V0f\ V1s))$

Definition 26 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0P)$

Definition 27 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2F)$.

Definition 28 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ (V0s\ V1t))$

Definition 29 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ (V0s\ V1t))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (c_2Erealax_2Etreall_lt\ T1\ T2)$

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal. (c_2Erealax_2Ereal_lt\ x\ y)$

Definition 32 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V0t1\ V1t2)))$

Definition 33 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ (V0x\ V1s))$

Definition 34 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_5C_2F\ A_27a)\ V0s)$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (28)$$

Definition 35 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(\lambda V1x \in A_27a.(ap$
Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A-27a})})^{((A_27b^{A_27b})^{A-27a})}) \end{aligned} \quad (29)$$

Definition 36 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2$

Definition 37 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^A-$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (30)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (31)$$

Definition 38 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A-27$

Definition 39 We define $c_2ELebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A-27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod\ (2^{2^{A-27a}}))\ (ty_2Erealax_2Ereal^{(2^{A-27a})})}) \end{aligned} \quad (32)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \end{aligned} \quad (33)$$

Definition 40 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \end{aligned} \quad (34)$$

Definition 41 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 42 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2$

Definition 43 We define $c_2E\text{lebesgue_2Epos_simple_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E$

Definition 44 We define $c_2E\text{lebesgue_2Epsfis}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E\text{prod } (2^{A-27a}) (ty_2E$

Definition 45 We define $c_2E\text{real_2Esup}$ to be $\lambda V0P \in (2^{ty_2E\text{realax_2E}E\text{real}}).(ap (c_2E\text{min_2E}E40 ty_2E\text{real}$

Definition 46 We define $c_2E\text{extreal_2Eextreal_sup}$ to be $\lambda V0p \in (2^{ty_2E\text{extreal_2E}E\text{extreal}}).(ap (ap (ap (c_2E$

Definition 47 We define $c_2E\text{lebesgue_2Epos_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E\text{prod } (2^{$

Definition 48 We define $c_2E\text{measure_2Efn_plus}$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2E\text{extreal_2E}E\text{extreal}^{A-27a}).$

Definition 49 We define $c_2E\text{pred_set_2EUNIV}$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2E\text{bool_2E}E\text{T}).$

Let $c_2E\text{measure_2E}E\text{subsets} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{measure_2E}E\text{subsets } A_27a \in ((2^{(2^{A-27a})})^{(ty_2E\text{pair_2E}E\text{prod } (2^{A-27a}) (2^{(2^{A-27a})}))}) \quad (35)$$

Definition 50 We define $c_2E\text{pred_set_2E}E\text{SUBSET}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Definition 51 We define $c_2E\text{pred_set_2E}E\text{INJ}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (2^{A-27a}).$

Definition 52 We define $c_2E\text{pred_set_2E}E\text{countable}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2E\text{bool_2E}E3F$

Definition 53 We define $c_2E\text{pred_set_2E}E\text{UNION}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c$

Let $c_2E\text{measure_2E}E\text{space} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{measure_2E}E\text{space } A_27a \in ((2^{A-27a})^{(ty_2E\text{pair_2E}E\text{prod } (2^{A-27a}) (2^{(2^{A-27a})}))}) \quad (36)$$

Definition 54 We define $c_2E\text{pred_set_2E}E\text{DIFF}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2E$

Definition 55 We define $c_2E\text{measure_2E}E\text{subset_class}$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})}).$

Definition 56 We define $c_2E\text{measure_2E}E\text{algebra}$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2E\text{pair_2E}E\text{prod } (2^{A-27a}) (2^{(2^{$

Definition 57 We define $c_2E\text{measure_2E}E\text{sigma_algebra}$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2E\text{pair_2E}E\text{prod } (2^{A-27a}) (2^{(2^{$

Definition 58 We define $c_2E\text{pred_set_2E}E\text{BIGINTER}$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2E\text{pred_set_2E}E$

Definition 59 We define $c_2E\text{measure_2E}E\text{sigma}$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap ($

Definition 60 We define $c_2E\text{measure_2E}E\text{Borel}$ to be $(ap (ap (c_2E\text{measure_2E}E\text{sigma } ty_2E\text{extreal_2E}E\text{extreal$

Definition 61 We define $c_2E\text{pred_set_2E}E\text{PREIMAGE}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V$

Definition 62 We define $c_2E\text{pred_set_2E}E\text{FUNSET}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in ($

Definition 63 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod$

Definition 64 We define $c_2Elebesgue_2Eintegrable$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a})$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum)}) \quad (37)$$

Definition 65 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 66 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 67 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 68 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 69 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECONJ$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (38)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (39)$$

Definition 70 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)}) \quad (40)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (41)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (42)$$

Definition 71 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$

Let $c_2Enets_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Eetends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (43)$$

Definition 72 We define $c_Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 73 We define c_Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 74 We define $c_Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 75 We define $c_Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A-27a}) (ty_2$

Definition 76 We define $c_Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A-27a}) (ty_2$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (47)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A-27a}).(\forall V1g \in (A_27b^{A-27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \quad (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\
& A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (((ap c.2Eextreal.2Eextreal_ainv c.2Eextreal.2ENegInf) = c.2Eextreal.2EPosInf) \wedge \\
& (((ap c.2Eextreal.2Eextreal_ainv c.2Eextreal.2EPosInf) = c.2Eextreal.2ENegInf) \wedge \\
& (\forall V0x \in ty.2Erealax.2Ereal.((ap c.2Eextreal.2Eextreal_ainv \\
& (ap c.2Eextreal.2ENormal V0x)) = (ap c.2Eextreal.2ENormal (ap \\
& c.2Erealax.2Ereal_neg V0x))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Eextreal.2Eextreal.(\forall V1y \in ty.2Eextreal.2Eextreal. \\
& ((ap (ap c.2Eextreal.2Eextreal_sub V0x) V1y) = (ap (ap c.2Eextreal.2Eextreal_add \\
& V0x) (ap c.2Eextreal.2Eextreal_ainv V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Eextreal.2Eextreal.((ap c.2Eextreal.2Eextreal_ainv \\
& V0x) = (ap (ap c.2Eextreal.2Eextreal_mul (ap c.2Eextreal.2Eextreal_ainv \\
& (ap c.2Eextreal.2Eextreal_of_num (ap c.2Earithmetic.2ENUMERAL \\
& (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))))) V0x)))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in (ty.2Epair.2Eprod \\
& (2^{A.27a}) (ty.2Epair.2Eprod (2^{(2^{A.27a})}) (ty.2Erealax.2Ereal^{(2^{A.27a})}))))). \\
& (\forall V1f1 \in (ty.2Eextreal.2Eextreal^{A.27a}).(\forall V2f2 \in \\
& (ty.2Eextreal.2Eextreal^{A.27a}).(((p (ap (c.2Emeasure.2Emeasure_space \\
& A.27a) V0m)) \wedge ((p (ap (ap (c.2Elebesgue.2Eintegrable A.27a) V0m) \\
& V1f1)) \wedge (p (ap (ap (c.2Elebesgue.2Eintegrable A.27a) V0m) V2f2)))))) \Rightarrow \\
& (p (ap (ap (c.2Elebesgue.2Eintegrable A.27a) V0m) (\lambda V3x \in A.27a. \\
& (ap (ap c.2Eextreal.2Eextreal_add (ap V1f1 V3x)) (ap V2f2 V3x)))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}).(\forall V2c \in ty_2Erealax_2Ereal. \\
& (((p\ (ap\ (c_2Emeasure_2Emeasure_space\ A.27a)\ V0m)) \wedge (p\ (ap\ (ap \\
& (c_2Elebesgue_2Eintegrable\ A.27a)\ V0m)\ V1f))) \Rightarrow (p\ (ap\ (ap\ (c_2Elebesgue_2Eintegrable \\
& A.27a)\ V0m)\ (\lambda V3x \in A.27a.(ap\ (ap\ c_2Eextreal_2Eextreal_mul \\
& (ap\ c_2Eextreal_2ENormal\ V2c))\ (ap\ V1f\ V3x)))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (74)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\ & (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\ & (\forall V1f1 \in (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V2f2 \in \\ & (ty_2Eextreal_2Eextreal^{A_27a}). (((p (ap (c_2Emeasure_2Emeasure_space \\ & A_27a) V0m)) \wedge ((p (ap (ap (c_2Elebesgue_2Eintegrable A_27a) V0m) \\ & V1f1)) \wedge (p (ap (ap (c_2Elebesgue_2Eintegrable A_27a) V0m) V2f2)))))) \Rightarrow \\ & (p (ap (ap (c_2Elebesgue_2Eintegrable A_27a) V0m) (\lambda V3x \in A_27a. \\ & (ap (ap c_2Eextreal_2Eextreal_sub (ap V1f1 V3x)) (ap V2f2 V3x)))))))))) \end{aligned}$$