

# thm\_2Elebesgue\_2Integral\_\_cmul (TMWDqe68EWfveo8T4qKAfMkXoGcdGTRo883)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p \text{ (ap } P \ x))$  **then**  $(\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o \ (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap \ V0P \ (ap \ (c_2Emin_2E_40 \ A_{27a} \ P) \ V0t) \ (ap \ (c_2Emin_2E_3D \ (2^{A_{27a}}) \ V0t) \ V0t)) \ (\lambda V1x \in 2.V0x)) \ (\lambda V1x \in 2.V0x))$

**Definition 4** We define `c_2Ebool_2ET` to be  $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V0x))$

**Definition 5** We define `c_2Ebool_2EBOUNDED` to be  $(\lambda V0v \in 2.c_2Ebool_2ET).$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Eextreal\_2Eextreal \tag{2}$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{3}$$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap \ (ap \ (c_2Emin_2E_3D \ (2^{A_{27a}}) \ V0t) \ V0t) \ (ap \ (c_2Emin_2E_3D \ (2^{A_{27a}}) \ V0t) \ V0t)) \ (\lambda V1x \in 2.V0x)) \ (\lambda V1x \in 2.V0x))$

**Definition 7** We define `c_2Ebool_2EF` to be  $(ap \ (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t)).$

**Definition 8** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o \ (p \ P \Rightarrow p \ Q)$  of type  $\iota.$

**Definition 9** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap \ (ap \ c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2EF \ V0t) \ (\lambda V1x \in 2.V0x))$

**Definition 10** We define `c_2Eextreal_2Eextreal__lt` to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextreal\_2Eextreal.$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \quad (4)$$

Let  $c\_2Eextreal\_2Eextreal\_sub : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_sub \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (5)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (6)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (7)$$

**Definition 11** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (8)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (10)$$

**Definition 12** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ c\_2Eextreal\_2ENormal\ c\_2Enum\_2E0)$ .

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (11)$$

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (c\_2Ebool\_2E\_21\ 2)\ t2)\ t1)\ t2)\ t1)$ .

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (c\_2Ebool\_2E\_21\ 2)\ t1\ t2)\ t1)\ t2)\ t1)$ .

**Definition 15** We define  $c\_2Emeasure\_2Efn\_minus$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a})$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (12)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))})) \quad (13)$$

**Definition 16** We define  $c\_Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (14)$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (15)$$

**Definition 18** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f\ V1s)$

**Definition 19** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0P\ V0P)$

**Definition 20** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF).$

**Definition 21** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0s\ V1t)$

**Definition 22** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0s\ V1t)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (16)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (17)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal)\ V0a\ V0a)$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (18)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Erealax\_2Etreal\_lt)\ T1\ T2)$

**Definition 25** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Erealax\_2Etreal\_lt)\ x\ y)$

**Definition 26** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ V0t1\ V1t2)))$

**Definition 27** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0x\ V1s)$

**Definition 28** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)\ V0s\ V0s)$

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in & ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \end{aligned} \quad (19)$$

**Definition 29** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (21)$$

**Definition 30** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 32** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 33** We define  $c\_2Emeasure\_2Eindicator\_fn$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (\lambda V1x \in A\_27a. (ap$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (23)$$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (24)$$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ & A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (25)$$

**Definition 34** We define  $c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal\_2Eextreal\_mul$

**Definition 35** We define  $c\_2Emeasure\_2Epos\_simple\_fn$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})$

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c.2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (26)$$
$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c.2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (27)$$

**Definition 37** We define c.Elebesgue\_2Epsfs to be  $\lambda A_{-27}a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A_{-27}}))\ (ty\_2L$

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c.2Emeasure\_2Emeasure\ A\_27a \in ( \\ & (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) \\ & \end{aligned} \quad (28)$$
$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (29)$$
$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal)})(30)$$
$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod \ ty\_2Ehreal\_2Ehreal \ ty\_2Ehreal\_2Ehreal)})}$$
(31)

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal) \\ (32)$$

**Definition 42** We define `c_2Elebesgue_2Epos__simple__fn__integral` to be  $\lambda A.27a : \iota.\lambda V0m \in (ty\_2Epair\_2E$

**Definition 43** We define  $c\_Elebesgue\_2Epsfis$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Ereal\_2Ereal))$

**Definition 44** We define  $c\_Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_Emin\_2E40 ty\_2Ereal\_2Ereal))$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \quad (33)$$

**Definition 45** We define  $c\_2Eextreal\_2Eextreal\_sup$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap (ap (ap (c\_2Eextreal\_2Eextreal\_sup))))$

**Definition 46** We define  $c\_Elebesgue\_2Epos\_fn\_integral$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Ereal\_2Ereal))$

**Definition 47** We define  $c\_2Emeasure\_2Efn\_plus$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a})$

**Definition 48** We define  $c\_Elebesgue\_2Eintegral$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Ereal\_2Ereal))$

**Definition 49** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2ET)$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A\_27a})}) (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})})) \quad (34)$$

**Definition 50** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Epred\_set\_2Ebool\_2ET))$

**Definition 51** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (A\_27b^{A\_27a}). (ap (c\_2Epred\_set\_2Ebool\_2ET))$

**Definition 52** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E3F))$

**Definition 53** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Epred\_set\_2Ebool\_2ET))$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))) \quad (35)$$

**Definition 54** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Epred\_set\_2Ebool\_2ET))$

**Definition 55** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A\_27a}). \lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 56** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Ereal\_2Ereal))$

**Definition 57** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Ereal\_2Ereal))$

**Definition 58** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap (c\_2Epred\_set\_2Ebool\_2ET))$

**Definition 59** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A\_27a}). \lambda V1st \in (2^{(2^{A\_27a})}). (ap (c\_2Emeasure\_2Esubset\_class))$

**Definition 60** We define  $c\_2Emeasure\_2EBorel$  to be  $(ap (ap (c\_2Emeasure\_2Esigma ty\_2Eextreal\_2Eextreal)))$

**Definition 61** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V$

**Definition 62** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A\_27a}). \lambda V1Q \in ($

**Definition 63** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod$

**Definition 64** We define  $c\_2Elebesgue\_2Eintegrable$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a})$

**Definition 65** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (36)$$

**Definition 66** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 67** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 68** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (37)$$

**Definition 69** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Ereal$

**Definition 70** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 71** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECON$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (38)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (39)$$

**Definition 72** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) \quad (40)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (41)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (42)$$

**Definition 73** We define  $c\_Emetric\_Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$   
 Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends \\ & A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b})})_{A\_27a})_{(A\_27a)^{A\_27b}}) \end{aligned} \quad (43)$$

**Definition 74** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 75** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 76** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 77** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2$

**Definition 78** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & \quad A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & \quad (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (50)$$



Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (58)$$

Assume the following.

$$(\forall V0v \in 2.((p \ (ap \ c\_2Ebool\_2EBOUNDED \ V0v)) \Leftrightarrow True)) \quad (59)$$

Assume the following.

$$(((ap \ c\_2Eextreal\_2Eextreal\_ainv \ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2EPosInf) \wedge (((ap \ c\_2Eextreal\_2Eextreal\_ainv \ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2ENegInf) \wedge (\forall V0x \in ty\_2Erealax\_2Ereal.((ap \ c\_2Eextreal\_2Eextreal\_ainv (ap \ c\_2Eextreal\_2ENormal \ V0x)) = (ap \ c\_2Eextreal\_2ENormal \ (ap \ c\_2Erealax\_2Ereal\_neg \ V0x)))))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealx\_2Ereal. ((\neg((ap\ c\_2Eextreal\_2ENormal \\
& V0x) = c\_2Eextreal\_2ENegInf)) \wedge (\neg((ap\ c\_2Eextreal\_2ENormal\ V0x) = \\
& c\_2Eextreal\_2EPosInf)))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\
& ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ c\_2Eextreal\_2ENegInf) \\
& (ap\ c\_2Eextreal\_2ENormal\ V1y))) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt \\
& (ap\ c\_2Eextreal\_2ENormal\ V1y))\ c\_2Eextreal\_2EPosInf)) \wedge (p\ ( \\
& ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf))) \wedge \\
& ((\neg(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ V0x)\ c\_2Eextreal\_2ENegInf))) \wedge \\
& ((\neg(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ c\_2Eextreal\_2EPosInf) \\
& V0x))) \wedge ((\neg(V0x = c\_2Eextreal\_2EPosInf)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt \\
& V0x)\ c\_2Eextreal\_2EPosInf))) \wedge ((\neg(V0x = c\_2Eextreal\_2ENegInf)) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ c\_2Eextreal\_2ENegInf)\ V0x))))))))) \\
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& (\forall V2z \in ty\_2Eextreal\_2Eextreal. (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt \\
& V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ V1y)\ V2z))) \Rightarrow ( \\
& p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ V0x)\ V2z)))))) \\
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& (((ap\ c\_2Eextreal\_2Eextreal\_ainv\ V0x) = (ap\ c\_2Eextreal\_2Eextreal\_ainv \\
& V1y)) \Leftrightarrow (V0x = V1y)))) \\
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& (((\neg(V0x = c\_2Eextreal\_2ENegInf)) \wedge (\neg(V0x = c\_2Eextreal\_2EPosInf))) \vee \\
& ((\neg(V1y = c\_2Eextreal\_2ENegInf)) \wedge (\neg(V1y = c\_2Eextreal\_2EPosInf)))) \Rightarrow \\
& ((ap\ c\_2Eextreal\_2Eextreal\_ainv\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_sub \\
& V0x)\ V1y)) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_sub\ V1y)\ V0x)))) \\
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& ((ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul\ (ap\ c\_2Eextreal\_2Eextreal\_ainv \\
& V0x))\ V1y) = (ap\ c\_2Eextreal\_2Eextreal\_ainv\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul \\
& V0x)\ V1y)))) \\
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& ((ap (ap c\_2Eextreal\_2Eextreal\_mul V0x) (ap c\_2Eextreal\_2Eextreal\_ainv \\
& V1y)) = (ap c\_2Eextreal\_2Eextreal\_ainv (ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& (\forall V2z \in ty\_2Eextreal\_2Eextreal. (((\neg(V0x = c\_2Eextreal\_2ENegInf)) \wedge \\
& ((\neg(V0x = c\_2Eextreal\_2EPosInf)) \wedge ((\neg(V1y = c\_2Eextreal\_2ENegInf)) \wedge \\
& ((\neg(V1y = c\_2Eextreal\_2EPosInf)) \wedge ((\neg(V2z = c\_2Eextreal\_2ENegInf)) \wedge \\
& (\neg(V2z = c\_2Eextreal\_2EPosInf)))))) \Rightarrow ((ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& V0x) (ap (ap c\_2Eextreal\_2Eextreal\_sub V1y) V2z)) = (ap (ap c\_2Eextreal\_2Eextreal\_sub \\
& (ap (ap c\_2Eextreal\_2Eextreal\_mul V0x) V1y)) (ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& V0x) V2z))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A\_27a) V0m)) \wedge (\forall V2x \in A\_27a. (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) (ap V1f V2x)))))) \Rightarrow \\
& (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap (c\_2Elebesgue\_2Epos\_fn\_integral A\_27a) \\
& V0m) V1f))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2c \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (c\_2Emeasure\_2Emeasure\_space A\_27a) V0m)) \wedge ((\forall V3x \in \\
& A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) (ap (c\_2Emeasure\_2Em\_space \\
& A\_27a) V0m))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum\_2E0)) (ap V1f V3x)))))) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2c)))) \Rightarrow ((ap (ap \\
& (c\_2Elebesgue\_2Epos\_fn\_integral A\_27a) V0m) (\lambda V4x \in A\_27a. \\
& (ap (ap c\_2Eextreal\_2Eextreal\_mul (ap c\_2Eextreal\_2ENormal \\
& V2c)) (ap V1f V4x)))) = (ap (ap c\_2Eextreal\_2Eextreal\_mul (ap c\_2Eextreal\_2ENormal \\
& V2c)) (ap (ap (c\_2Elebesgue\_2Epos\_fn\_integral A\_27a) V0m) V1f))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\ & (\forall V1x \in A.27a.(p\ (ap\ (ap\ c.2Eextreal\_2Eextreal\_le\ (ap\ c.2Eextreal\_2Eextreal\_of\_num \\ & \quad c.2Enum\_2E0))\ (ap\ (ap\ (c.2Emeasure\_2Efn\_plus\ A.27a)\ V0g)\ V1x)))))) \\ & \hspace{15em} (71) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\ & (\forall V1x \in A.27a.(p\ (ap\ (ap\ c.2Eextreal\_2Eextreal\_le\ (ap\ c.2Eextreal\_2Eextreal\_of\_num \\ & \quad c.2Enum\_2E0))\ (ap\ (ap\ (c.2Emeasure\_2Efn\_minus\ A.27a)\ V0g)\ V1x)))))) \\ & \hspace{15em} (72) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\ & (\forall V1c \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte \\ & (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0))\ V1c)) \Rightarrow ((ap\ (c.2Emeasure\_2Efn\_plus \\ & \quad A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ c.2Eextreal\_2Eextreal\_mul\ (ap \\ & \quad c.2Eextreal\_2ENormal\ V1c))\ (ap\ V0f\ V2x)))))) = (\lambda V3x \in A.27a.( \\ & \quad ap\ (ap\ c.2Eextreal\_2Eextreal\_mul\ (ap\ c.2Eextreal\_2ENormal\ V1c)) \\ & \quad (ap\ (ap\ (c.2Emeasure\_2Efn\_plus\ A.27a)\ V0f)\ V3x)))))) \wedge ((p\ (ap\ ( \\ & \quad ap\ c.2Ereal\_2Ereal\_lte\ V1c)\ (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0))) \Rightarrow \\ & ((ap\ (c.2Emeasure\_2Efn\_plus\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Eextreal\_2Eextreal\_mul \\ & \quad (ap\ c.2Eextreal\_2ENormal\ V1c))\ (ap\ V0f\ V4x)))))) = (\lambda V5x \in A.27a. \\ & \quad (ap\ (ap\ c.2Eextreal\_2Eextreal\_mul\ (ap\ c.2Eextreal\_2Eextreal\_ainv \\ & \quad (ap\ c.2Eextreal\_2ENormal\ V1c))\ (ap\ (ap\ (c.2Emeasure\_2Efn\_minus \\ & \quad \quad A.27a)\ V0f)\ V5x)))))))))) \\ & \hspace{15em} (73) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\ & (\forall V1c \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c.2Ereal\_2Ereal\_lte \\ & (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0))\ V1c)) \Rightarrow ((ap\ (c.2Emeasure\_2Efn\_minus \\ & \quad A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ c.2Eextreal\_2Eextreal\_mul\ (ap \\ & \quad c.2Eextreal\_2ENormal\ V1c))\ (ap\ V0f\ V2x)))))) = (\lambda V3x \in A.27a.( \\ & \quad ap\ (ap\ c.2Eextreal\_2Eextreal\_mul\ (ap\ c.2Eextreal\_2ENormal\ V1c)) \\ & \quad (ap\ (ap\ (c.2Emeasure\_2Efn\_minus\ A.27a)\ V0f)\ V3x)))))) \wedge ((p\ (ap\ ( \\ & \quad ap\ c.2Ereal\_2Ereal\_lte\ V1c)\ (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0))) \Rightarrow \\ & ((ap\ (c.2Emeasure\_2Efn\_minus\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap \\ & \quad c.2Eextreal\_2Eextreal\_mul\ (ap\ c.2Eextreal\_2ENormal\ V1c))\ ( \\ & \quad ap\ V0f\ V4x)))))) = (\lambda V5x \in A.27a.(ap\ (ap\ c.2Eextreal\_2Eextreal\_mul \\ & \quad (ap\ c.2Eextreal\_2Eextreal\_ainv\ (ap\ c.2Eextreal\_2ENormal\ V1c)) \\ & \quad (ap\ (ap\ (c.2Emeasure\_2Efn\_plus\ A.27a)\ V0f)\ V5x)))))))))) \\ & \hspace{15em} (74) \end{aligned}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y))))))) \quad (75)$$

Assume the following.

$$((ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \quad (76)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))))) \quad (77)$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte V0y) V1x)))))) \quad (78)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (79)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (80)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (84)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{88}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{92}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{93}$$

### Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal(2^{A.27a}))))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). (\forall V2c \in ty\_2Erealax\_2Ereal. \\
& (((p \ (ap \ (c\_2Emeasure\_2Emeasure\_space \ A.27a) \ V0m)) \wedge (p \ (ap \ (ap \\
& (c\_2Elebesgue\_2Eintegrable \ A.27a) \ V0m) \ V1f))) \Rightarrow ((ap \ (ap \ (c\_2Elebesgue\_2Eintegral \\
& A.27a) \ V0m) \ (\lambda V3x \in A.27a. (ap \ (ap \ c\_2Eextreal\_2Eextreal\_mul \\
& (ap \ c\_2Eextreal\_2ENormal \ V2c)) \ (ap \ V1f \ V3x)))))) = (ap \ (ap \ c\_2Eextreal\_2Eextreal\_mul \\
& (ap \ c\_2Eextreal\_2ENormal \ V2c)) \ (ap \ (ap \ (c\_2Elebesgue\_2Eintegral \\
& A.27a) \ V0m) \ V1f))))))
\end{aligned}$$