

# thm\_2Elebesgue\_2Integral\_\_cmul\_\_indicator (TMb5V3hqTukJvdgAdzCdfScGgXUa4vqNimU)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a})).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P))$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c})).\lambda V1g \in (A\_27c^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) f)) g$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p (ap\ P\ x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (ty\_2Erealax\_2Ereal\_mul) a))$ .  
Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$ .

**Definition 11** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (8)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (13)$$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \quad (14)$$



Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in & ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \end{aligned} \quad (21)$$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (22)$$

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (23)$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (24)$$

**Definition 25** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 26** We define  $c\_2Eextreal\_2Eextreal\_lt$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextreal$

**Definition 27** We define  $c\_2Emeasure\_2Efn\_minus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a})$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in \\ & ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \end{aligned} \quad (25)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (26)$$

**Definition 28** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E\_2C\ x\ y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (27)$$

**Definition 29** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in A\_27b$

**Definition 30** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ P))))$

**Definition 31** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EBIGUNION\ P))$

**Definition 32** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 33** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21) 2)$

**Definition 34** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21) 2)$

**Definition 35** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 36** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21) 2) (\lambda V2t \in 2)))$

**Definition 37** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21) 2)$

**Definition 38** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E21) 2)$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (28)$$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EITSET A\_27a A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (29)$$

**Definition 39** We define  $c\_2Eextreal\_2EEEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal\_2Eextreal)$

**Definition 40** We define  $c\_2Emeasure\_2Epos\_simple\_fn$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Erealax\_2Ereal))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (30)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (31)$$

**Definition 41** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27a})^{A\_27b})$

**Definition 42** We define  $c\_2ELebesgue\_2Epsfs$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Erealax\_2Ereal))$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)^{ty\_2Ehreal\_2Ehreal})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (32)$$

**Definition 43** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 44** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2E)$

**Definition 45** We define  $c\_2Elebesgue\_2Epos\_simple\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2E)$

**Definition 46** We define  $c\_2Elebesgue\_2Epsfis$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2E))$

**Definition 47** We define  $c\_2Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Emin\_2E40 ty\_2Ereal))$

**Definition 48** We define  $c\_2Eextreal\_2Eextreal\_sup$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap (ap (ap (c\_2E))))$

**Definition 49** We define  $c\_2Elebesgue\_2Epos\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2E))$

**Definition 50** We define  $c\_2Emeasure\_2Efn\_plus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A-27a}).$

Let  $c\_2Eextreal\_2Eextreal\_sub : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_sub \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (33)$$

**Definition 51** We define  $c\_2Elebesgue\_2Eintegral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2E))$

**Definition 52** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A-27a})})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))} \quad (34)$$

**Definition 53** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Ebool\_2E3F))$

**Definition 54** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V1s \in (2^{A-27a})$

**Definition 55** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E3F))$

**Definition 56** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Ebool\_2E3F))$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod (2^{A-27a}) (2^{(2^{A-27a})}))}) \quad (35)$$

**Definition 57** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_2Ebool\_2E3F))$

**Definition 58** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

**Definition 59** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2E))$

**Definition 60** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2E))$

**Definition 61** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c\_2Epred\_s$

**Definition 62** We define  $c\_2Emeasure\_2Esigma$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap ($

**Definition 63** We define  $c\_2Emeasure\_2EBorel$  to be  $(ap (ap (c\_2Emeasure\_2Esigma ty\_2Eextreal\_2Eextre$

**Definition 64** We define  $c\_2Epred\_set\_2EPREIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V$

**Definition 65** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in ($

**Definition 66** We define  $c\_2Emeasure\_2Emeasurable$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod$

**Definition 67** We define  $c\_2Elebesgue\_2Eintegrable$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A-27a})$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod ty\_2Eenum\_2Eenum)})(36)$$

**Definition 68** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 69** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 70** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(37)$$

**Definition 71** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

**Definition 72** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 73** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) (38)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}})(39)$$

**Definition 74** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})) \quad (40)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (41)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (42)$$

**Definition 75** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A-27b})^{A-27b})}))_{A\_27a}(A\_27a^{A-27b})) \quad (43)$$

**Definition 76** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 77** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 78** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 79** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A-27a})\ (ty\_2$

**Definition 80** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^A$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (47)$$



Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ (\forall V3y \in A\_27a.((ap V0f (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ V1b) V2x) V3y)) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V1b) (ap V0f \\ V2x)) (ap V0f V3y)))))) \quad (52) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(((ap (ap (c\_2Ecombin\_2Eo A\_27a A\_27b \\ A\_27b) (c\_2Ecombin\_2EI A\_27b)) V0f) = V0f) \wedge ((ap (ap (c\_2Ecombin\_2Eo \\ A\_27a A\_27b A\_27a) V0f) (c\_2Ecombin\_2EI A\_27a)) = V0f))) \quad (54) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((ap (ap c\_2Eextreal\_2Eextreal\_mul c\_2Eextreal\_2ENegInf) \\
c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2EPosInf) \wedge (((ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& \quad c\_2Eextreal\_2ENegInf) c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2ENegInf) \wedge \\
& \quad ((ap (ap c\_2Eextreal\_2Eextreal\_mul c\_2Eextreal\_2EPosInf) \\
c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf) \wedge (((ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& \quad c\_2Eextreal\_2EPosInf) c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2EPosInf) \wedge \\
& \quad (((ap (ap c\_2Eextreal\_2Eextreal\_mul (ap c\_2Eextreal\_2ENormal \\
V0x)) c\_2Eextreal\_2ENegInf) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0))) (ap c\_2Eextreal\_2ENormal (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
V0x)) c\_2Eextreal\_2ENegInf) c\_2Eextreal\_2EPosInf))) \wedge ((ap \\
& \quad (ap c\_2Eextreal\_2Eextreal\_mul c\_2Eextreal\_2ENegInf) (ap c\_2Eextreal\_2ENormal \\
V1y)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) ( \\
& \quad ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) V1y) (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0))) (ap c\_2Eextreal\_2ENormal (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
V1y)) c\_2Eextreal\_2ENegInf) c\_2Eextreal\_2EPosInf))) \wedge ((ap \\
& \quad (ap c\_2Eextreal\_2Eextreal\_mul (ap c\_2Eextreal\_2ENormal V0x)) \\
c\_2Eextreal\_2EPosInf) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0))) (ap c\_2Eextreal\_2ENormal (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
V0x)) c\_2Eextreal\_2EPosInf) c\_2Eextreal\_2ENegInf))) \wedge ((ap \\
& \quad (ap c\_2Eextreal\_2Eextreal\_mul c\_2Eextreal\_2EPosInf) (ap c\_2Eextreal\_2ENormal \\
V1y)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) ( \\
& \quad ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) V1y) (ap c\_2Ereal\_2Ereal\_of\_num \\
c\_2Enum\_2E0))) (ap c\_2Eextreal\_2ENormal (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Eextreal\_2Eextreal) \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
V1y)) c\_2Eextreal\_2EPosInf) c\_2Eextreal\_2ENegInf))) \wedge ((ap ( \\
& \quad ap c\_2Eextreal\_2Eextreal\_mul (ap c\_2Eextreal\_2ENormal V0x)) \\
(ap c\_2Eextreal\_2ENormal V1y)) = (ap c\_2Eextreal\_2ENormal (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)))))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1s \in (2^{A.27a}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A.27a) V0m)) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a}) V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a) V0m)))) \Rightarrow (p (ap (ap (c\_2Elebesgue\_2Eintegrable A.27a) V0m) \\
& (ap (c\_2Emeasure\_2Eindicator\_fn A.27a) V1s))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1s \in (2^{A.27a}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A.27a) V0m)) \wedge (p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a}) V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a) V0m)))) \Rightarrow ((ap (ap (c\_2Elebesgue\_2Eintegral A.27a) V0m) \\
& (ap (c\_2Emeasure\_2Eindicator\_fn A.27a) V1s)) = (ap c\_2Eextreal\_2ENormal \\
& (ap (ap (c\_2Emeasure\_2Emeasure A.27a) V0m) V1s))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V2c \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (c\_2Emeasure\_2Emeasure\_space A.27a) V0m)) \wedge (p (ap (ap \\
& (c\_2Elebesgue\_2Eintegrable A.27a) V0m) V1f))) \Rightarrow ((ap (ap (c\_2Elebesgue\_2Eintegral \\
& A.27a) V0m) (\lambda V3x \in A.27a.(ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& (ap c\_2Eextreal\_2ENormal V2c) (ap V1f V3x)))) = (ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& (ap c\_2Eextreal\_2ENormal V2c) (ap (ap (c\_2Elebesgue\_2Eintegral \\
& A.27a) V0m) V1f))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in \\ & 2.(((p V0p) \Leftrightarrow (p (ap (ap (ap (c.2Ebool.2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\ & (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\ & (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg( \\ & p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (74)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\ & (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\ & (\forall V1s \in (2^{A.27a}).(\forall V2c \in ty\_2Erealax\_2Ereal.(( \\ & (p (ap (c\_2Emeasure\_2Emeasure\_space \ A.27a) \ V0m)) \wedge (p (ap (ap ( \\ & c\_2Ebool\_2EIN (2^{A.27a}) \ V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\ & A.27a) \ V0m)))))) \Rightarrow ((ap (ap (c\_2Elebesgue\_2Eintegral \ A.27a) \ V0m) \\ & (\lambda V3x \in A.27a.(ap (ap \ c.2Eextreal\_2Eextreal\_mul (ap \ c.2Eextreal\_2ENormal \\ & V2c)) (ap (ap (c\_2Emeasure\_2Eindicator\_fn \ A.27a) \ V1s) \ V3x)))) = \\ & (ap \ c.2Eextreal\_2ENormal (ap (ap \ c.2Erealax\_2Ereal\_mul \ V2c) \\ & (ap (ap (c\_2Emeasure\_2Emeasure \ A.27a) \ V0m) \ V1s)))))) \end{aligned}$$