

# thm\_2Elebesgue\_2Integral\_mono (TMTJtSZUXh3f3rXdFAZxGYBAoioy5AQcsgn)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

**Definition 4** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ecombin\_2E\_2S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 7** We define  $c\_2Ecombin\_2E\_2I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2S A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P)))$

**Definition 9** We define  $c\_2Ecombin\_2E\_2o$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27a}))$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Let  $c\_2Eextreal\_2Eextreal\_sub : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_sub \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{3}$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal}) \tag{4}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))})) \quad (7)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (8)$$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (9)$$

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (10)$$

**Definition 14** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in 2. (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f\ V1s)$

**Definition 15** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ V0P)$

**Definition 16** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2. V0t))$ .

**Definition 17** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 18** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27a)\ V0s\ V1t)$

**Definition 19** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27a)\ V0s\ V1t)$

**Definition 20** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E3D\_3D\_3E V0t) c\_Ebool\_E7E))$

Let  $c\_EEnum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_EEnum\_EZERO\_REP \in \omega \tag{11}$$

Let  $c\_EEnum\_EABS\_num : \iota$  be given. Assume the following.

$$c\_EEnum\_EABS\_num \in (ty\_EEnum\_EEnum^{\omega}) \tag{12}$$

**Definition 21** We define  $c\_EEnum\_E0$  to be  $(ap c\_EEnum\_EABS\_num c\_EEnum\_EZERO\_REP)$ .

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealax\_Ereal^{ty\_EEnum\_EEnum}) \tag{13}$$

Let  $ty\_Ehreal\_Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Ehreal\_Ehreal \tag{14}$$

Let  $c\_Erealax\_Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{ty\_Erealax\_Ereal}) \tag{15}$$

**Definition 22** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap (c\_Emin\_E40 (t$

Let  $c\_Erealax\_Etrealm : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)}) \tag{16}$$

**Definition 23** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 24** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$

**Definition 25** We define  $c\_Ebool\_E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21 2) (\lambda V2t \in 2$

**Definition 26** We define  $c\_Epred\_set\_EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_Ebool\_E21 2)$

**Definition 27** We define  $c\_Epred\_set\_EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_Ebool\_E21 2)$

Let  $c\_Emeasure\_Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Emeasure\_Emeasurable\_sets \\ & A\_27a \in (((2^{(2^{A\_27a})})^{(ty\_Epair\_Eprod\ (2^{A\_27a})\ (ty\_Epair\_Eprod\ (2^{(2^{A\_27a})})\ (ty\_Erealax\_Ereal^{(2^{A\_27a})}))})) \end{aligned} \tag{17}$$

Let  $c\_Eextreal\_ENormal : \iota$  be given. Assume the following.

$$c\_Eextreal\_ENormal \in (ty\_Eextreal\_Eextreal^{ty\_Erealax\_Ereal}) \tag{18}$$

**Definition 28** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal\_2Eextreal\_of\_num)$

**Definition 29** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (20)$$

**Definition 30** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (21)$$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT1))$

**Definition 32** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 33** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ c\_2Ebool\_2ECOND))))$

**Definition 34** We define  $c\_2Emeasure\_2Eindicator\_fn$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(\lambda V1x \in A\_27a.(ap\ c\_2Emeasure\_2Eindicator\_fn))$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal} \quad (22)$$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (23)$$

**Definition 35** We define  $c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal)$

**Definition 36** We define  $c\_2Emeasure\_2Epos\_simple\_fn$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a}))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (24)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (25)$$

**Definition 37** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

**Definition 38** We define  $c\_2Elebesgue\_2Epsfs$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2E$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Emeasure A\_27a \in ( (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})) (26)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (27)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (28)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} (29)$$

**Definition 39** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 40** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (30)$$

**Definition 41** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 42** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal$

**Definition 43** We define  $c\_2Elebesgue\_2Epos\_simple\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 44** We define  $c\_2Elebesgue\_2Epsfs$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2E$

**Definition 45** We define  $c\_2Ereal\_2Esup$  to be  $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Emin\_2E.40 ty\_2Ereal$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal (31)$$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal (32)$$

**Definition 46** We define  $c\_2Eextreal\_2Eextreal\_sup$  to be  $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap (ap (ap (c\_2E$

**Definition 47** We define  $c\_2ELebesgue\_2Epos\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{$

Let  $c\_2Eextreal\_2Eextreal\_ainv : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (33)$$

**Definition 48** We define  $c\_2Eextreal\_2Eextreal\_lt$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

**Definition 49** We define  $c\_2Emeasure\_2Efn\_minus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}$

**Definition 50** We define  $c\_2Emeasure\_2Efn\_plus$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).$

**Definition 51** We define  $c\_2ELebesgue\_2Eintegral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a} (ty$

**Definition 52** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EET).$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod ty\_2Eenum\_2Eenum)}) \quad (34)$$

**Definition 53** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 54** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

**Definition 55** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (35)$$

**Definition 56** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

**Definition 57** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 58** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECON$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (36)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \quad (37)$$

**Definition 59** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealx\_2Ereal) (ap (c\_2Emetric\_2Edist : \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealx\_2Ereal)^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}) \quad (38)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (39)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^{A\_27a})})}) \quad (40)$$

**Definition 60** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A\_27b})^{A\_27b}))})_{A\_27a})_{(A\_27a)^{A\_27b}})) \quad (41)$$

**Definition 61** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal)^{ty\_2Enum\_2Enum}.\lambda V1x$

**Definition 62** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal)^{ty\_2Enum\_2Enum}.\lambda V1s \in ty\_2$

**Definition 63** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A\_27b})$

**Definition 64** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 65** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (ty\_2$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Esubsets A\_27a \in (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))} \quad (42)$$

**Definition 66** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

**Definition 67** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b)^{A\_27a}.\lambda V1s \in (2^{A\_27a})$

**Definition 68** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E3F$

**Definition 69** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emeasure\_2Espace A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))}) \quad (43)$$

**Definition 70** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 71** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 72** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 73** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

**Definition 74** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^{A\_27a}) (2^{(2^{A\_27a})}))$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (49)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (50)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (52)$$



Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0b \in 2. (\forall V1f \in (A\_27b^{A\_27a}). (\forall V2g \in (A\_27b^{A\_27a}). \\
& (\forall V3x \in A\_27a. (((ap (ap (ap (ap (c\_2Ebool\_2ECOND (A\_27b^{A\_27a})) \\
& V0b) V1f) V2g) V3x) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V0b) (ap \\
& V1f V3x)) (ap V2g V3x))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}). (\forall V1b \in 2. (\forall V2x \in A\_27a. \\
& (\forall V3y \in A\_27a. (((ap V0f (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\
& V1b) V2x) V3y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V1b) (ap V0f \\
& V2x)) (ap V0f V3y))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\
& V5y\_27))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI \\
& A\_27a) V0x) = V0x))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\ & \quad A\_27b)\ (c\_2Ecombin\_2Ei\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\ & \quad A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2Ei\ A\_27a)) = V0f))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul \\ & \quad V0x)\ (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num \\ & \quad c\_2Enum\_2E0))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.((ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul \\ & \quad V0x)\ (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) = V0x)) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & \quad V0x)\ V0x))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. \\ & \quad ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & \quad V0x)\ V1y)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. \\ & \quad (\forall V2z \in ty\_2Eextreal\_2Eextreal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & \quad V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ V1y)\ V2z))) \Rightarrow ( \\ & \quad p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ V0x)\ V2z)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. \\ & \quad ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ (ap\ c\_2Eextreal\_2Eextreal\_ainv \\ & \quad V0x))\ (ap\ c\_2Eextreal\_2Eextreal\_ainv\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & \quad V1y)\ V0x)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. \\ & \quad ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ (ap\ c\_2Eextreal\_2Eextreal\_ainv \\ & \quad V0x))\ (ap\ c\_2Eextreal\_2Eextreal\_ainv\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt \\ & \quad V1y)\ V0x)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty\_2Eextreal\_2Eextreal. (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
& (\forall V2y \in ty\_2Eextreal\_2Eextreal. (\forall V3z \in ty\_2Eextreal\_2Eextreal. \\
& (((p (ap (ap c\_2Eextreal\_2Eextreal\_le V0w) V1x)) \wedge (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& V2y) V3z)))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap (ap c\_2Eextreal\_2Eextreal\_add \\
& V0w) V2y)) (ap (ap c\_2Eextreal\_2Eextreal\_add V1x) V3z)))))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (\forall V1y \in ty\_2Eextreal\_2Eextreal. \\
& ((ap (ap c\_2Eextreal\_2Eextreal\_sub V0x) V1y) = (ap (ap c\_2Eextreal\_2Eextreal\_add \\
& V0x) (ap c\_2Eextreal\_2Eextreal\_ainv V1y))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Eextreal\_2Eextreal\_ainv (ap c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum\_2E0)) = (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2g \in ( \\
& ty\_2Eextreal\_2Eextreal^{A\_27a}). ((\forall V3x \in A\_27a. ((p (ap ( \\
& ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum\_2E0)) (ap V1f V3x))) \wedge (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& (ap V1f V3x)) (ap V2g V3x)))))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& (ap (ap (c\_2Elebesgue\_2Epos\_fn\_integral A\_27a) V0m) V1f)) ( \\
& ap (ap (c\_2Elebesgue\_2Epos\_fn\_integral A\_27a) V0m) V2g)))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A\_27a}) (ty\_2Epair\_2Eprod (2^{(2^{A\_27a})}) (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). ((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A\_27a) V0m)) \Rightarrow ((ap (ap (c\_2Elebesgue\_2Eintegral A\_27a) V0m) V1f) = \\
& (ap (ap (c\_2Elebesgue\_2Eintegral A\_27a) V0m) (\lambda V2x \in A\_27a. \\
& (ap (ap c\_2Eextreal\_2Eextreal\_mul (ap V1f V2x)) (ap (ap (c\_2Emeasure\_2Eindicator\_fn \\
& A\_27a) (ap (c\_2Emeasure\_2Em\_space A\_27a) V0m)) V2x)))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\
& (\forall V1x \in A\_27a. (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap (c\_2Emeasure\_2Efn\_plus A\_27a) V0g) V1x))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0g \in (ty\_2Eextreal\_2Eextreal^{A_{27a}}). \\ & (\forall V1x \in A_{27a}. (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap c\_2Eextreal\_2Eextreal\_of\_num \\ & c\_2Enum\_2E0)) (ap (ap (c\_2Emeasure\_2Efn\_minus A_{27a}) V0g) V1x)))))) \end{aligned} \quad (74)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (76)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (78)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (79)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in \\ & 2.(((p V0p) \Leftrightarrow (p (ap (ap (ap (c.2Ebool.2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\ & (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\ & (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))))) \quad (85) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (88)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (89)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (90)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\ & (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealx\_2Ereal^{(2^{A.27a})}))). \\ & (\forall V1f1 \in (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V2f2 \in \\ & (ty\_2Eextreal\_2Eextreal^{A.27a}).(((p (ap (c.2Emeasure\_2Emeasure\_space \\ & A.27a) V0m)) \wedge (\forall V3t \in A.27a.((p (ap (ap (c.2Ebool\_2EIN A.27a) \\ & V3t) (ap (c.2Emeasure\_2Em\_space A.27a) V0m))) \Rightarrow (p (ap (ap c.2Eextreal\_2Eextreal\_le \\ & (ap V1f1 V3t) (ap V2f2 V3t)))))) \Rightarrow (p (ap (ap c.2Eextreal\_2Eextreal\_le \\ & (ap (ap (c.2Elebesgue\_2Eintegral A.27a) V0m) V1f1) (ap (ap (c.2Elebesgue\_2Eintegral \\ & A.27a) V0m) V2f2))))))))) \end{aligned}$$