

thm_2ELebesgue_2Eintegral__mspace (TMcvFpd3neFfdzcdBjBanAWFG6dQHQJFzre)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y) \text{ of type } \iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ P)))$

Definition 4 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ecombin_2E_2S` to be $\lambda A. \lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 6 We define `c_2Ecombin_2E_2K` to be $\lambda A. \lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 7 We define `c_2Ecombin_2E_2I` to be $\lambda A. \lambda A_27a : \iota. (\text{ap } (\text{ap } (c_2Ecombin_2E_2S \ A_27a \ (A_27a^{A_27a})) \ A_27a))$

Definition 8 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ P))$

Definition 9 We define `c_2Ecombin_2E_2o` to be $\lambda A. \lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}).$

Let `ty_2Eextreal_2Eextreal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Eextreal_2Eextreal \tag{1}$$

Let `c_2Eextreal_2Eextreal_sub` : ι be given. Assume the following.

$$c_2Eextreal_2Eextreal_sub \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Let `c_2Enum_2EZERO_REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \tag{3}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{4}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{5}$$

Definition 19 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}})^{A_27a}) \end{aligned} \quad (13)$$

Definition 20 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (14)$$

Definition 21 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in A_27b. (ap\ (c_2Epair_2EABS_prod\ f\ s))$

Definition 22 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ P))$

Definition 23 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 24 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epair_2EABS_prod\ s\ t))$

Definition 25 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epair_2EABS_prod\ s\ t))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (15)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal_REP_CLASS}) \quad (16)$$

Definition 26 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal. (ap\ (c_2Emin_2E40\ a))$

Let $c_2Erealx_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 27 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal. \lambda V1T2 \in ty_2Erealx_2Ereal. (ap\ (c_2Emin_2E40\ T1\ T2))$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal. \lambda V1y \in ty_2Erealx_2Ereal. (ap\ (c_2Emin_2E40\ x\ y))$

Definition 29 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$

Definition 30 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epair_2EABS_prod\ x\ s))$

Definition 31 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ s)$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in & ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealx_2Ereal^{(2^{A_27a})})))}) \end{aligned} \quad (18)$$

Definition 32 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (20)$$

Definition 33 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 34 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 35 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 36 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (22)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (23)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in & (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \quad (24)$$

Definition 37 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal$

Definition 38 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (25)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (26)$$

Definition 39 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Definition 40 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})))$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal)^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a}))} (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})) (ty_2Erealax_2Ereal)^{(2^{A_27a})} \end{aligned} \quad (27)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (28)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (29)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (30)$$

Definition 41 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 42 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (31)$$

Definition 43 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 44 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal)$

Definition 45 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})))$

Definition 46 We define $c_2E\text{lebesgue_2Epsfis}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A_27a}) (ty_2E\text{real_2Ereal}))$

Definition 47 We define $c_2E\text{real_2Esup}$ to be $\lambda V0P \in (2^{ty_2E\text{realax_2Ereal}}).(ap (c_2E\text{min_2E.40 } ty_2E\text{real_2Ereal}))$

Let $c_2E\text{extreal_2ENegInf} : \iota$ be given. Assume the following.

$$c_2E\text{extreal_2ENegInf} \in ty_2E\text{extreal_2Eextreal} \quad (32)$$

Let $c_2E\text{extreal_2EPosInf} : \iota$ be given. Assume the following.

$$c_2E\text{extreal_2EPosInf} \in ty_2E\text{extreal_2Eextreal} \quad (33)$$

Definition 48 We define $c_2E\text{extreal_2Eextreal_sup}$ to be $\lambda V0p \in (2^{ty_2E\text{extreal_2Eextreal}}).(ap (ap (ap (c_2E\text{min_2E.40 } ty_2E\text{real_2Ereal})))$

Definition 49 We define $c_2E\text{lebesgue_2Epos_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A_27a}) (ty_2E\text{real_2Ereal}))$

Definition 50 We define $c_2E\text{measure_2Efn_plus}$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2E\text{extreal_2Eextreal}^{A_27a})$.

Definition 51 We define $c_2E\text{lebesgue_2Eintegral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2Eprod } (2^{A_27a}) (ty_2E\text{real_2Ereal}))$

Definition 52 We define $c_2E\text{pred_set_2EUNIV}$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2E\text{bool_2ET})$.

Let $c_2E\text{real_2Esum} : \iota$ be given. Assume the following.

$$c_2E\text{real_2Esum} \in ((ty_2E\text{realax_2Ereal}^{(ty_2E\text{realax_2Ereal}^{ty_2E\text{enum_2Eenum}})})^{(ty_2E\text{pair_2Eprod } ty_2E\text{enum_2Eenum})}) \quad (34)$$

Definition 53 We define $c_2E\text{prim_rec_2E.3C}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Definition 54 We define $c_2E\text{arithmic_2E.3E}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Definition 55 We define $c_2E\text{arithmic_2E.3E.3D}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Let $c_2E\text{realax_2Etrealm_neg} : \iota$ be given. Assume the following.

$$c_2E\text{realax_2Etrealm_neg} \in ((ty_2E\text{pair_2Eprod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})^{(ty_2E\text{pair_2Eprod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})}) \quad (35)$$

Definition 56 We define $c_2E\text{realax_2Ereal_neg}$ to be $\lambda V0T1 \in ty_2E\text{realax_2Ereal}.(ap c_2E\text{realax_2Ereal})$

Definition 57 We define $c_2E\text{real_2Ereal_sub}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.\lambda V1y \in ty_2E\text{realax_2Ereal}$

Definition 58 We define $c_2E\text{real_2Eabs}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.(ap (ap (ap (c_2E\text{bool_2ECONJ})))$

Let $ty_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2E\text{metric_2Emetric } A0) \quad (36)$$

Let $c_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{metric_2Emetric } A_27a \in ((ty_2E\text{metric_2Emetric } A_27a)^{(ty_2E\text{realax_2Ereal}^{(ty_2E\text{pair_2Eprod } A_27a } A_27a)})}) \quad (37)$$

Definition 70 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 71 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 72 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 73 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Definition 74 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge (((\neg False) \Leftrightarrow True)))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (52)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b^{A_27a})\ V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V0b)\ (ap\ V1f\ V3x))\ (ap\ V2g\ V3x))))))) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f\ V2x))\ (ap\ V0f\ V3y))))))) \quad (60)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \Rightarrow (61)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{27a}.(\forall V3x_{27} \in A_{27a}.(\forall V4y \in A_{27a}.(\forall V5y_{27} \in A_{27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{27})))))) \Rightarrow ((ap (ap (ap (c_{2Ebool}_{2ECOND} A_{27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{2Ebool}_{2ECOND} A_{27a}) V1Q) V3x_{27}) V5y_{27})))))) \Rightarrow (62)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_{2Ecombin}_{2EI} A_{27a}) V0x) = V0x)) \Rightarrow (63)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(((ap (ap (c_{2Ecombin}_{2Eo} A_{27a} A_{27b}) V0f) = V0f) \wedge ((ap (ap (c_{2Ecombin}_{2EI} A_{27a} A_{27b}) V0f) = V0f) \wedge ((ap (ap (c_{2Ecombin}_{2EI} A_{27a}) V0f) = V0f)))))) \Rightarrow (64)$$

Assume the following.

$$(\forall V0x \in ty_{2Eextreal}_{2Eextreal}.((ap (ap c_{2Eextreal}_{2Eextreal_mul} V0x) (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0})) = (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0}))) \Rightarrow (65)$$

Assume the following.

$$(\forall V0x \in ty_{2Eextreal}_{2Eextreal}.((ap (ap c_{2Eextreal}_{2Eextreal_mul} (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0})) V0x) = (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0}))) \Rightarrow (66)$$

Assume the following.

$$(\forall V0x \in ty_{2Eextreal}_{2Eextreal}.((ap (ap c_{2Eextreal}_{2Eextreal_mul} V0x) (ap c_{2Eextreal}_{2Eextreal_of_num} (ap c_{2Earithmetic}_{2ENUMERAL} (ap c_{2Earithmetic}_{2EBIT1} c_{2Earithmetic}_{2EZERO})))) = V0x)) \Rightarrow (67)$$

Assume the following.

$$((ap c_{2Eextreal}_{2Eextreal_ainv} (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0})) = (ap c_{2Eextreal}_{2Eextreal_of_num} c_{2Enum}_{2E0})) \Rightarrow (68)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A-27a}) (ty_2Epair_2Eprod (2^{(2^{A-27a})}) (ty_2Erealx_2Ereal^{(2^{A-27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A-27a}).(((p (ap (c_2Emeasure_2Emeasure_space \\
& A.27a) V0m)) \wedge (\forall V2x \in A.27a.(p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) (ap V1f V2x)))))) \Rightarrow \\
& ((ap (ap (c_2Elebesgue_2Epos_fn_integral\ A.27a) V0m) V1f) = \\
& (ap (ap (c_2Elebesgue_2Epos_fn_integral\ A.27a) V0m) (\lambda V3x \in \\
& A.27a.(ap (ap c_2Eextreal_2Eextreal_mul (ap V1f V3x)) (ap (ap \\
& (c_2Emeasure_2Eindicator_fn\ A.27a) (ap (c_2Emeasure_2Em_space \\
& A.27a) V0m)) V3x)))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A-27a}). \\
& (\forall V1x \in A.27a.(p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap (ap (c_2Emeasure_2Efn_plus\ A.27a) V0g) V1x))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A-27a}). \\
& (\forall V1x \in A.27a.(p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap (ap (c_2Emeasure_2Efn_minus\ A.27a) V0g) V1x))))))
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (ap (ap (ap (c_2Ebool.2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V3s)))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg \\
& p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{82}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{87}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\ & (2^{A_{.27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{.27a}})}) (ty_2Erealx_2Ereal(2^{A_{.27a}}))))). \\ & (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}).((p (ap (c_2Emeasure_2Emeasure_space \\ & A_{.27a}) V0m)) \Rightarrow ((ap (ap (c_2Elebesgue_2Eintegral A_{.27a}) V0m) V1f) = \\ & (ap (ap (c_2Elebesgue_2Eintegral A_{.27a}) V0m) (\lambda V2x \in A_{.27a}. \\ & (ap (ap c_2Eextreal_2Eextreal_mul (ap V1f V2x)) (ap (ap (c_2Emeasure_2Eindicator_fn \\ & A_{.27a}) (ap (c_2Emeasure_2Em_space A_{.27a}) V0m)) V2x)))))))))) \end{aligned}$$