

thm_2ELebesgue_2Eintegral__zero
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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2. V 0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2. V 2t))))$

Definition 7 We define `c_2Ebool_2E_7E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t) \text{ c_2Ebool_2E_2F}))))$

Let `ty_2Eextreal_2Eextreal` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eextreal_2Eextreal} \tag{1}$$

Let `c_2Eextreal_2Eextreal__sub` : ι be given. Assume the following.

$$\text{c_2Eextreal_2Eextreal_sub} \in ((\text{ty_2Eextreal_2Eextreal}^{\text{ty_2Eextreal_2Eextreal}})^{\text{ty_2Eextreal_2Eextreal}}) \tag{2}$$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$\text{c_2Enum_2EZERO_REP} \in \text{omega} \tag{3}$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{4}$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EABS_num} \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \tag{5}$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(14)

Definition 16 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 20 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 21 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(15)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS})$$
(16)

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 ($

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(17)

Definition 23 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 26 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 27 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))}))$$
(18)

Definition 28 We define $c_2\text{Earithmetic_EZERO}$ to be $c_2\text{Enum_E0}$.

Let $c_2\text{Enum_EREP_num} : \iota$ be given. Assume the following.

$$c_2\text{Enum_EREP_num} \in (\text{omega}^{ty_2\text{Enum_Enum}}) \quad (19)$$

Let $c_2\text{Enum_ESUC_REP} : \iota$ be given. Assume the following.

$$c_2\text{Enum_ESUC_REP} \in (\text{omega}^{\text{omega}}) \quad (20)$$

Definition 29 We define $c_2\text{Enum_ESUC}$ to be $\lambda V0m \in ty_2\text{Enum_Enum} . (ap\ c_2\text{Enum_EABS_num}$

Let $c_2\text{Earithmetic_E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_E_2B} \in ((ty_2\text{Enum_Enum}^{ty_2\text{Enum_Enum}})_{ty_2\text{Enum_Enum}}) \quad (21)$$

Definition 30 We define $c_2\text{Earithmetic_EBIT1}$ to be $\lambda V0n \in ty_2\text{Enum_Enum} . (ap\ (ap\ c_2\text{Earithmetic}$

Definition 31 We define $c_2\text{Earithmetic_ENUMERAL}$ to be $\lambda V0x \in ty_2\text{Enum_Enum} . V0x$.

Definition 32 We define $c_2\text{Emeasure_Eindicator_fn}$ to be $\lambda A_27a : \iota . \lambda V0s \in (2^{A_27a}) . (\lambda V1x \in A_27a . (ap$

Let $c_2\text{Eextreal_Eextreal_mul} : \iota$ be given. Assume the following.

$$c_2\text{Eextreal_Eextreal_mul} \in ((ty_2\text{Eextreal_Eextreal}^{ty_2\text{Eextreal_Eextreal}})_{ty_2\text{Eextreal_Eextreal}}) \quad (22)$$

Let $c_2\text{Eextreal_Eextreal_add} : \iota$ be given. Assume the following.

$$c_2\text{Eextreal_Eextreal_add} \in ((ty_2\text{Eextreal_Eextreal}^{ty_2\text{Eextreal_Eextreal}})_{ty_2\text{Eextreal_Eextreal}}) \quad (23)$$

Let $c_2\text{Epred_set_EITSET} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a . \text{nonempty } A_27a \Rightarrow \forall A_27b . \text{nonempty } A_27b \Rightarrow c_2\text{Epred_set_EITSET} \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \quad (24)$$

Definition 33 We define $c_2\text{Eextreal_EEXTREAL_SUM_IMAGE}$ to be $\lambda A_27a : \iota . \lambda V0f \in (ty_2\text{Eextreal_Eextreal}$

Definition 34 We define $c_2\text{Emeasure_Epos_simple_fn}$ to be $\lambda A_27a : \iota . \lambda V0m \in (ty_2\text{Epair_Eprod } (2^{A_27a}$

Let $c_2\text{Epair_ESND} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a . \text{nonempty } A_27a \Rightarrow \forall A_27b . \text{nonempty } A_27b \Rightarrow c_2\text{Epair_ESND} \\ A_27a\ A_27b \in (A_27b^{(ty_2\text{Epair_Eprod } A_27a\ A_27b)}) \end{aligned} \quad (25)$$

Let $c_2\text{Epair_EFST} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a . \text{nonempty } A_27a \Rightarrow \forall A_27b . \text{nonempty } A_27b \Rightarrow c_2\text{Epair_EFST} \\ A_27a\ A_27b \in (A_27a^{(ty_2\text{Epair_Eprod } A_27a\ A_27b)}) \end{aligned} \quad (26)$$

Definition 35 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 36 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})) (27)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (28)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (29)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} (30)$$

Definition 37 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 38 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (31)$$

Definition 39 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 40 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Definition 41 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 42 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 43 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40 ty_2Ereal$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal (32)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal (33)$$

Definition 44 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 45 We define $c_2Elebesgue_2Epos_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{$

Definition 46 We define $c_2Emeasure_2Efn_plus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a}).$

Definition 47 We define $c_2Elebesgue_2Eintegral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 48 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EET).$

Definition 49 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum)}) \quad (34)$$

Definition 50 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 51 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 52 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (35)$$

Definition 53 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Definition 54 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 55 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECON$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (36)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (37)$$

Definition 56 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})) \quad (38)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (39)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (40)$$

Definition 57 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A-27b})^{A-27b})}))_{A_27a}(A_27a^{A-27b})) \quad (41)$$

Definition 58 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 59 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 60 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in (2^{A-27b})$

Definition 61 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 62 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A-27a})\ (ty_2Epair_2Eprod$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A-27a})})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^A-27a)})})) \quad (42)$$

Definition 63 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

Definition 64 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in (2^{A-27a})$

Definition 65 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E_3F$

Definition 66 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2Ebool_2E_3F$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ (2^{A-27a})\ (2^{(2^A-27a)})})) \quad (43)$$

Definition 67 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 68 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a}}$

Definition 69 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a}}$

Definition 70 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a}}$

Definition 71 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a}}$

Assume the following.

$$True \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in 2.(\forall V1t \in A_27a.((ap (ap (ap (c_2Ebool_2ECOND A_27a) V0b) V1t) V1t) = V1t))) \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (52)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \Rightarrow \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A.27a.(\forall V3x.27 \in A.27a.(\forall V4y \in A.27a. \\ & (\forall V5y.27 \in A.27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x.27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y.27)))))) \Rightarrow ((ap (ap (ap (c.2Ebool.2ECOND A.27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool.2ECOND A.27a) V1Q) V3x.27) \\ & V5y.27))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0t1 \in A.27a.(\forall V1t2 \in \\ & A.27a.((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A.27a.(\forall V3t2 \in A.27a.((ap \\ & (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF) V2t1) V3t2) = V3t2)))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0x \in ty.2Eextreal.2Eextreal.(\neg(p (ap (ap c.2Eextreal.2Eextreal_lt V0x) V0x)))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Eextreal.2Eextreal.((ap (ap c.2Eextreal.2Eextreal_sub \\ & (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)) V0x) = (ap c.2Eextreal.2Eextreal_ainv \\ & V0x))) \end{aligned} \quad (57)$$

Assume the following.

$$((ap c.2Eextreal.2Eextreal_ainv (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)) = (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0)) \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0m \in (ty.2Epair.2Eprod \\ & (2^{A.27a}) (ty.2Epair.2Eprod (2^{(2^{A.27a})}) (ty.2Erealax.2Ereal^{(2^{A.27a})}))). \\ & ((p (ap (c.2Emeasure.2Emeasure_space A.27a) V0m)) \Rightarrow ((ap (ap (\\ & c.2Elebesgue.2Epos_fn_integral A.27a) V0m) (\lambda V1x \in A.27a. \\ & (ap c.2Eextreal.2Eextreal_of_num c.2Enum.2E0))) = (ap c.2Eextreal.2Eextreal_of_num \\ & c.2Enum.2E0)))) \end{aligned} \quad (59)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V_0 m \in (ty_2Epair_2Eprod \\ & (2^{A_{27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{27a}})}) (ty_2Erealx_2Ereal^{(2^{A_{27a}})}))) \\ & ((p (ap (c_2Emeasure_2Emeasure_space\ A_{27a})\ V_0 m)) \Rightarrow ((ap (ap (\\ c_2Elebesgue_2Eintegral\ A_{27a})\ V_0 m) (\lambda V_1 x \in A_{27a}.(ap\ c_2Eextreal_2Eextreal_of_num \\ c_2Enum_2E0))) = (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)))) \end{aligned}$$