

thm_2Elebesgue_2Elemma_fn_5 (TMWEEx- tjtFNRMdNkNSKVdN27UFeEvcdbjUj)

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Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \tag{2}$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \tag{3}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{4}$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \tag{5}$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal})^{ty_2Eextreal_2Eextreal}) \tag{6}$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{7}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.\lambda \tau a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a})))$

Definition 4 We define c_Ebool_2E to be $(ap (c_Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E))$

Definition 7 We define $c_Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal_2Eextreal$

Let $c_2Eextreal_2Eextreal_inv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_inv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \quad (8)$$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (9)$$

Definition 8 We define $c_Eextreal_2Eextreal_div$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal_2Eextreal$

Definition 9 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 11 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_Ebool_2E_3F$ to be $\lambda A^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_Emin_2E_40))))$

Definition 13 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (14)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (15)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (16)$$

Definition 15 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Epred_set_2EGSPEC\ n\ n))$

Let $c_2Eextreal_2Eextreal_pow : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_pow \in ((ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum})^{ty_2Eextreal_2Eextreal}) \quad (17)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (18)$$

Definition 16 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_pow\ n)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (19)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (21)$$

Definition 21 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 22 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_Earithmetic_2EBIT1\ V0n))$

Definition 23 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ V2t2\ V1t1))))$

Definition 24 We define $c_Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (\lambda V1x \in A_27a. (ap\ V1x\ V0s))$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (23)$$

Definition 25 We define $c_Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Extreal_2Extreal)$

Definition 26 We define $c_Elebesgue_2Efn_seq$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Enum_2Enum))$

Let $c_Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (24)$$

Let $c_Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (25)$$

Definition 27 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ V1n\ V0m)$

Definition 28 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ V1t2\ V0t1)))$

Definition 29 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ V1n\ V0m)$

Definition 30 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_Ebool_2E_21\ 2)\ V0m)))$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Let $c_Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 31 We define $c_Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (28)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \quad (29)$$

Definition 32 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (t$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (30)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (31)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (32)$$

Definition 33 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 34 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (33)$$

Definition 35 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 36 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum})_{ty_2Erealax_2Ereal} \quad (34)$$

Definition 37 We define $c_2Earithmic_2E3C_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (35)$$

Definition 38 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 39 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)\ (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)) \quad (36)$$

Definition 40 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal.$

Assume the following.

$$(\forall V0n \in ty_Eenum_Eenum.(p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ c_Eenum_E0)\ V0n))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Eenum_Eenum.(\forall V1n \in ty_Eenum_Eenum.(\\ & ((ap\ (ap\ c_Earithmetic_E_2A\ c_Eenum_E0)\ V0m) = c_Eenum_E0) \wedge \\ & (((ap\ (ap\ c_Earithmetic_E_2A\ V0m)\ c_Eenum_E0) = c_Eenum_E0) \wedge \\ & (((ap\ (ap\ c_Earithmetic_E_2A\ (ap\ c_Earithmetic_E_ENUMERAL \\ & (ap\ c_Earithmetic_E_BIT1\ c_Earithmetic_E_ZERO)))\ V0m) = V0m) \wedge \\ & (((ap\ (ap\ c_Earithmetic_E_2A\ V0m)\ (ap\ c_Earithmetic_E_ENUMERAL \\ & (ap\ c_Earithmetic_E_BIT1\ c_Earithmetic_E_ZERO))) = V0m) \wedge \\ & ((ap\ (ap\ c_Earithmetic_E_2A\ (ap\ c_Eenum_E_ESUC\ V0m))\ V1n) = (ap \\ & (ap\ c_Earithmetic_E_2B\ (ap\ (ap\ c_Earithmetic_E_2A\ V0m)\ V1n)) \\ & V1n)) \wedge ((ap\ (ap\ c_Earithmetic_E_2A\ V0m)\ (ap\ c_Eenum_E_ESUC\ V1n)) = \\ & (ap\ (ap\ c_Earithmetic_E_2B\ V0m)\ (ap\ (ap\ c_Earithmetic_E_2A \\ & V0m)\ V1n)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Eenum_Eenum.(\forall V1n \in ty_Eenum_Eenum.(\\ & \forall V2p \in ty_Eenum_Eenum.(((p\ (ap\ (ap\ c_Earithmetic_E_3C_3D \\ & V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V1n)\ V2p))) \Rightarrow (p\ (\\ & ap\ (ap\ c_Earithmetic_E_3C_3D\ V0m)\ V2p)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Eenum_Eenum.(\forall V1n \in ty_Eenum_Eenum.(\\ & \forall V2p \in ty_Eenum_Eenum.(((p\ (ap\ (ap\ c_Earithmetic_E_3C_3D \\ & (ap\ (ap\ c_Earithmetic_E_2B\ V0m)\ V1n))\ (ap\ (ap\ c_Earithmetic_E_2B \\ & V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V1n)\ V2p)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Eenum_Eenum.(\forall V1n \in ty_Eenum_Eenum.(\\ & (\neg(p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D \\ & (ap\ c_Eenum_E_ESUC\ V1n))\ V0m)))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0n))) \quad (42)$$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (49)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ & \quad ap\ V1Q\ V4x))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & \quad A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\ & \quad V2x))\ (ap\ V0f\ V3y))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2a \in ty_2Eextreal_2Eextreal. (\forall V3v2 \in ty_2Erealax_2Ereal. \\
& (\forall V4v3 \in ty_2Erealax_2Ereal. (\forall V5v5 \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2ENegInf) \\
& V2a)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V3v2)) c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V4v3)) c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& (ap c_2Eextreal_2ENormal V5v5))) \Leftrightarrow False)))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Eextreal_2Eextreal_pow (ap c_2Eextreal_2ENormal \\
& V0a)) V1n) = (ap c_2Eextreal_2ENormal (ap (ap c_2Ereal_2Epow V0a) \\
& V1n)))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Eextreal_2Eextreal_pow \\
& c_2Eextreal_2EPosInf) V2n) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V2n) c_2Enum_2E0)) (ap \\
& c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) c_2Eextreal_2EPosInf))) \wedge \\
& ((\forall V3n \in ty_2Enum_2Enum. ((ap (ap c_2Eextreal_2Eextreal_pow \\
& c_2Eextreal_2ENegInf) V3n) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V3n) c_2Enum_2E0)) (ap \\
& c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) (ap (ap \\
& (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) (ap c_2Earithmetic_2EVEN \\
& V3n)) c_2Eextreal_2EPosInf) c_2Eextreal_2ENegInf))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& V0x) V1y))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (p (ap (ap c_2Eextreal_2Eextreal_le \\
& V0x) V0x)))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& ((p (ap (ap (ap c_2Eextreal_2Eextreal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le \\
& V0x) V1y))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Eextreal_2Eextreal_div (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y)) = (ap c_2Eextreal_2ENormal \\
& (ap (ap c_2Ereal_2E_2F V0x) V1y))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3x \in A_27a. (\forall V4k \in ty_2Enum_2Enum. ((p (ap (ap \\
& (c_2Ebool_2EIN A_27a) V3x) (ap (c_2Emeasure_2Em_space A_27a) \\
& V0m))) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V4k) (ap c_2Epred_set_2Ecount \\
& (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V2n)))) \wedge ((p (ap (ap c_2Eextreal_2Eextreal_le (ap (ap c_2Eextreal_2Eextreal_div \\
& (ap c_2Eextreal_2Eextreal_of_num V4k)) (ap (ap c_2Eextreal_2Eextreal_pow \\
& (ap c_2Eextreal_2Eextreal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) V2n))) \\
& (ap V1f V3x))) \wedge (p (ap (ap c_2Eextreal_2Eextreal_lt (ap V1f V3x)) \\
& (ap (ap c_2Eextreal_2Eextreal_div (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap c_2Eextreal_2Eextreal_of_num V4k)) (ap c_2Eextreal_2Eextreal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& (ap (ap c_2Eextreal_2Eextreal_pow (ap c_2Eextreal_2Eextreal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) \\
& V2n)))))) \Rightarrow ((ap (ap (ap (ap (c_2ELebesgue_2Efn_seq A_27a) V0m) \\
& V1f) V2n) V3x) = (ap (ap c_2Eextreal_2Eextreal_div (ap c_2Eextreal_2Eextreal_of_num \\
& V4k)) (ap (ap c_2Eextreal_2Eextreal_pow (ap c_2Eextreal_2Eextreal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) \\
& V2n)))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3x \in A.27a. ((p (ap (ap (c_2Ebool_2EIN\ A.27a)\ V3x) (ap \\
& (c_2Emeasure_2Em_space\ A.27a)\ V0m))) \wedge (p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap (ap\ c_2Eextreal_2Eextreal_pow (ap\ c_2Eextreal_2Eextreal_of_num \\
& (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& V2n)) (ap\ V1f\ V3x)))) \Rightarrow ((ap (ap (ap (ap (c_2ELebesgue_2Efn_seq \\
& A.27a)\ V0m)\ V1f)\ V2n)\ V3x) = (ap (ap\ c_2Eextreal_2Eextreal_pow \\
& (ap\ c_2Eextreal_2Eextreal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) V2n)))))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3x \in A.27a. ((p (ap (ap (c_2Ebool_2EIN\ A.27a)\ V3x) (ap \\
& (c_2Emeasure_2Em_space\ A.27a)\ V0m))) \wedge (p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) (ap\ V1f\ V3x)))) \Rightarrow \\
& ((p (ap (ap\ c_2Eextreal_2Eextreal_le (ap (ap\ c_2Eextreal_2Eextreal_pow \\
& (ap\ c_2Eextreal_2Eextreal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) V2n)) (\\
& ap\ V1f\ V3x))) \vee (\exists V4k \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Enum_2Enum)\ V4k) (ap\ c_2Epred_set_2Ecount (ap (ap\ c_2Earithmetic_2EEXP \\
& (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT2 (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) V2n)))) \wedge ((p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap (ap\ c_2Eextreal_2Eextreal_div (ap\ c_2Eextreal_2Eextreal_of_num \\
& V4k)) (ap (ap\ c_2Eextreal_2Eextreal_pow (ap\ c_2Eextreal_2Eextreal_of_num \\
& (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& V2n)) (ap\ V1f\ V3x))) \wedge (p (ap (ap\ c_2Eextreal_2Eextreal_lt (ap \\
& V1f\ V3x)) (ap (ap\ c_2Eextreal_2Eextreal_div (ap (ap\ c_2Eextreal_2Eextreal_add \\
& (ap\ c_2Eextreal_2Eextreal_of_num\ V4k)) (ap\ c_2Eextreal_2Eextreal_of_num \\
& (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\
& (ap (ap\ c_2Eextreal_2Eextreal_pow (ap\ c_2Eextreal_2Eextreal_of_num \\
& (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) \\
& V2n))))))))) \\
& \hspace{15em} (68)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty_2Epair_2Eprod} \\
& (2^{A_{27a}}) (\text{ty_2Epair_2Eprod } (2^{(2^{A_{27a}})}) (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})}))) \\
& (\forall V1f \in (\text{ty_2Eextreal_2Eextreal}^{A_{27a}}). (\forall V2n \in \text{ty_2Enum_2Enum} \\
& (\forall V3x \in A_{27a}. ((\neg(p (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } A_{27a}) V3x) (\text{ap} \\
& (\text{c_2Emeasure_2Em_space } A_{27a}) V0m)))))) \Rightarrow ((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Elebesgue_2Efn_seq} \\
& A_{27a}) V0m) V1f) V2n) V3x) = (\text{ap } \text{c_2Eextreal_2Eextreal_of_num} \\
& \text{c_2Enum_2E0}))))))
\end{aligned}
\tag{69}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y))) \Rightarrow (\neg (V0x = V1y))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F V0x) V1y))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& ((p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \Rightarrow (p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Ereal_2Epow V0x) V1n))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0n)) (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V0n))) (ap c_2Ereal_2Ereal_of_num \\
& V1m))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0n)) (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num \\
& V1m)))) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge ((p (ap (ap \\
& c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num \\
& V0n)) (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num \\
& V1m)))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D V1m) V0n))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& V0n)) (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V0n))) (ap c_2Ereal_2Ereal_of_num \\
& V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge (((\\
& p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& V0n)) (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num \\
& V1m)))) \Leftrightarrow False) \wedge ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V0n))) (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n))))))
\end{aligned} \tag{78}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.((p \vee 0A) \Rightarrow ((\neg(p \vee 0A)) \Rightarrow \text{False}))) \quad (80)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \vee 0A) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \vee 0A)) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \vee 0A)) \Rightarrow \text{False}) \Rightarrow (((p \vee 0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))))))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \wedge (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \vee (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Rightarrow (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))) \quad (88)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p \ V0p) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ 2) \ V1q) \ V2r) \ V3s))) \Leftrightarrow \\
& (((p \ V0p) \vee ((p \ V1q) \vee (\neg(p \ V3s)))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge \\
& (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V3s)))) \wedge (((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))) \wedge ((p \ V1q) \vee ((p \ V3s) \vee (\neg(p \ V0p))))))))))))) \\
& \tag{89}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{92}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{93}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{94}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) \ (ty_2Epair_2Eprod \ (2^{(2^{A_27a})}) \ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3x \in A_27a. ((p \ (ap \ (ap \ c_2Eextreal_2Eextreal_le \ (ap \\
& \ c_2Eextreal_2Eextreal_of_num \ c_2Enum_2E0)) \ (ap \ V1f \ V3x))) \Rightarrow \\
& (p \ (ap \ (ap \ c_2Eextreal_2Eextreal_le \ (ap \ c_2Eextreal_2Eextreal_of_num \\
& \ c_2Enum_2E0)) \ (ap \ (ap \ (ap \ (ap \ (c_2Elebesgue_2Efn_seq \ A_27a) \ V0m) \\
& \ V1f) \ V2n) \ V3x)))))))))
\end{aligned}$$