thm_2Elebesgue_2Elemma__fn__upper__bounded (TMJyTFCb4i3JHP1hp3EyZsHcz392Pmr2HyU)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (1)$$

Let $c\_2Eextreal\_2Eextreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (2)$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (3)$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eextr$

Let $c\_2Eextreal\_2Eextreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_inv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \quad (4)$$

Let $c\_2Eextreal\_2Eextreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (5)$$

Definition 8 *We define* c_2Eextreal_2Eextreal__div *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{7}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{8}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ $(\mathsf{c\_2Ebool\_2E\_21}\ 2)$ $(\lambda V2t \in$

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ $(\mathsf{c\_2Emin\_2E\_40}$

Definition 15 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{13}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{14}$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{15}$$

Definition 17 *We define* c_2Epred__set_2Ecount *to be* $\lambda V0n\in ty\_2Enum\_2Enum.(ap\ (c\_2Epred\_\_set\_2EG$

Definition 18 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n\in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x\in ty\_2Enum\_2Enum.V0x$*.*

Let $c\_2Eextreal\_2Eextreal\_\_pow:\iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_pow\in((ty\_2Eextreal\_2Eextreal^{ty\_2Enum\_2Enum})^{ty\_2Eextreal\_2Eextreal}) \tag{16}$$

Let $ty\_2Erealax\_2Ereal:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{17}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num:\iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num\in(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{18}$$

Let $c\_2Eextreal\_2ENormal:\iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENormal\in(ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \tag{19}$$

Definition 21 *We define* c_2Eextreal_2Eextreal__of__num *to be* $\lambda V0n\in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

Let $c\_2Emeasure\_2Em\_\_space:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Emeasure\_2Em\_\_space\ A\_27a\in((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{20}$$

Definition 22 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t\in 2.(\lambda V1t1\in A\_27a.(\lambda V2t2\in A\_27a.($

Definition 24 *We define* c_2Emeasure_2Eindicator__fn *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(\lambda V1x\in A\_27a.(ap$

Let $c\_2Epred\_\_set\_2EITSET:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EITSET\ A\_27a\ A\_27b\in(((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{21}$$

Definition 25 *We define* c_2Eextreal_2EEXTREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2$

Definition 26 *We define* c_2Elebesgue_2Efn__seq *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty$

Definition 27 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{27}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land ((\neg False) \Leftrightarrow True))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum. (\forall V3x \in A\_27a.(\forall V4k \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V4k)\ (ap\ \mathsf{c\_2Epred\_\_set\_2Ecount}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n)))) \wedge ((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_div}\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ V4k))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n)))\ (ap\ V1f\ V3x))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ (ap\ V1f\ V3x))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_div}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ V4k))\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))))))) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Efn\_\_seq}\ A\_27a)\ V0m)\ V1f)\ V2n)\ V3x) = (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_div}\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ V4k))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3x \in A\_27a.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)))\land(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))\ (ap\ V1f\ V3x))))\Rightarrow((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Efn\_\_seq}\ A\_27a)\ V0m)\ V1f)\ V2n)\ V3x) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3x \in A\_27a.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)))\land(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ V1f\ V3x))))\Rightarrow((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))\ (ap\ V1f\ V3x)))\lor(\exists V4k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V4k)\ (ap\ \mathsf{c\_2Epred\_\_set\_2Ecount}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))))\land((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_div}\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ V4k))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n)))\ (ap\ V1f\ V3x)))\land(p\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_lt}\ (ap\ V1f\ V3x))\ (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_div}\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ V4k))\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_pow\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2n))))))))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3x \in A\_27a.((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)))) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Efn\_\_seq}\ A\_27a)\ V0m)\ V1f)\ V2n)\ V3x) = (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{51}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2n \in ty\_2Enum\_2Enum.(\forall V3x \in A\_27a.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num\ c\_2Enum\_2E0}))\ (ap\ V1f\ V3x)))\Rightarrow(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Efn\_\_seq}\ A\_27a)\ V0m)\ V1f)\ V2n)\ V3x))\ (ap\ V1f\ V3x))))))))$$