

# thm\_2Elebesgue\_2Elemma\_radon\_max\_in\_F (TMVkdLrovq8rqEVKrmKx9eUxnutnxJr38Sn)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P))$

**Definition 8** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27a^{A\_27c}).$

Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let  $c\_2Eextreal\_2ENegInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{2}$$

Let  $c\_2Eextreal\_2EPosInf : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{3}$$

Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal)^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal} \tag{4}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (6)$$

Let  $c\_2Eextreal\_2Eextreal\_le : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (7)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))) \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

**Definition 9** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ V1t2))))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (11)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (12)$$

**Definition 13** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in 2. (ap\ V1s\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0f))$

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ 2)\ V0P))))$

**Definition 15** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap (c\_2Epred\_set\_2EBIGUNION) P)$ .

**Definition 16** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 17** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 18** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Epred\_set\_2EINTER) s t)$ .

**Definition 19** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Epred\_set\_2EDISJOINT) s t)$ .

**Definition 20** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E) t)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{13}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{14}$$

**Definition 21** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{15}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{16}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{17}$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap (c\_2Emin\_2E\_40) a)$ .

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \tag{18}$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal. (ap (c\_2Erealax\_2Ereal\_lt) T1 T2)$ .

**Definition 24** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal. (ap (c\_2Ereal\_2Ereal\_lte) x y)$ .

**Definition 25** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_5C\_2F) t1 t2) t2)))$ .

**Definition 26** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A-27a}). (ap (c\_2Epred\_set\_2EINSERT) x s)$ .

**Definition 27** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap (c\_2Ebool\_2E\_21) 2)$ .

Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets \\ A\_27a \in & ((2^{(2^{A\_27a})}) (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})))) \end{aligned} \quad (19)$$

**Definition 28** We define  $c\_2Eextreal\_2Eextreal\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

**Definition 29** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (21)$$

**Definition 30** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 32** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 33** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 34** We define  $c\_2Emeasure\_2Eindicator\_fn$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (\lambda V1x \in A\_27a. (ap$

Let  $c\_2Eextreal\_2Eextreal\_mul : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (23)$$

Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a\ A\_27b \in & (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (24)$$

**Definition 35** We define  $c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Eextreal$

**Definition 36** We define  $c\_2Emeasure\_2Epos\_simple\_fn$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^A$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (25)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (26)$$

**Definition 37** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a})$

**Definition 38** We define  $c\_2Elebesgue\_2Epsfs$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{A\_27a})))$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ( \\ (ty\_2Erealax\_2Ereal)^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{2^{A\_27a}}))\ (ty\_2Erealax\_2Ereal)^{(2^{A\_27a}}))} \end{aligned} \quad (27)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (28)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (29)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (30)$$

**Definition 39** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 40** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (31)$$

**Definition 41** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 42** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal)$

**Definition 43** We define  $c\_2Elebesgue\_2Epos\_simple\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{A\_27a})))$





**Definition 74** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$   
 Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends \\ & A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b})})_{A\_27a})_{(A\_27a)^{A\_27b}}) \end{aligned} \quad (41)$$

**Definition 75** We define  $c\_2Eseq\_2E\_2D\_2D\_2E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 76** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2E$

**Definition 77** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 78** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2E$

**Definition 79** We define  $c\_2Emeasure\_2Emeasure\_space$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2E$

**Definition 80** We define  $c\_2Eextreal\_2Eextreal\_max$  to be  $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2E$

Assume the following.

$$True \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \wedge (p\ V1t2)) \Leftrightarrow ((p\ V1t2) \wedge (p\ V0t1)))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (48)$$



Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (50)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (52)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (54)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (55)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\
& V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\exists V1x \in \\ A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ 2^{A\_27a}). (((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ 2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ( \\ (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (63)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (64)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ ((\neg(p\ V1Q) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ V5y\_27)))))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (67)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))))) \quad (68)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (69)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (((ap (ap (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap (ap (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f)))) \quad (70)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & (\forall V2a \in ty\_2Eextreal\_2Eextreal. (\forall V3v2 \in ty\_2Erealx\_2Ereal. \\ & (\forall V4v5 \in ty\_2Erealx\_2Ereal. (\forall V5v3 \in ty\_2Erealx\_2Ereal. \\ & (((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ (ap\ c\_2Eextreal\_2ENormal\ V1y)) = (ap\ c\_2Eextreal\_2ENormal\ (ap\ (ap\ c\_2Erealx\_2Ereal\_add\ V0x)\ V1y))) \wedge (((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ c\_2Eextreal\_2EPosInf)\ V2a) = c\_2Eextreal\_2EPosInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2EPosInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ c\_2Eextreal\_2ENegInf)\ (ap\ c\_2Eextreal\_2EPosInf)) = c\_2Eextreal\_2EPosInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ c\_2Eextreal\_2ENegInf)\ (ap\ c\_2Eextreal\_2ENegInf)) = c\_2Eextreal\_2ENegInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ c\_2Eextreal\_2ENegInf)\ (ap\ c\_2Eextreal\_2ENegInf)) = c\_2Eextreal\_2ENegInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ (ap\ c\_2Eextreal\_2ENormal\ V4v5)) = c\_2Eextreal\_2ENegInf) \wedge ((ap (ap\ c\_2Eextreal\_2Eextreal\_add\ (ap\ c\_2Eextreal\_2ENormal\ V5v3))\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf)))))))))) \quad (71) \end{aligned}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap (ap\ c\_2Eextreal\_2Eextreal\_mul\ V0x)\ (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0))) \quad (72)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap (ap c\_2Eextreal\_2Eextreal\_mul \\
& V0x) (ap c\_2Eextreal\_2Eextreal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) = V0x))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& V0x) V0x)))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty\_2Eextreal\_2Eextreal. (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
& (\forall V2y \in ty\_2Eextreal\_2Eextreal. (\forall V3z \in ty\_2Eextreal\_2Eextreal. \\
& (((p (ap (ap c\_2Eextreal\_2Eextreal\_le V0w) V1x)) \wedge (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& V2y) V3z)))) \Rightarrow (p (ap (ap c\_2Eextreal\_2Eextreal\_le (ap (ap c\_2Eextreal\_2Eextreal\_add \\
& V0w) V2y)) (ap (ap c\_2Eextreal\_2Eextreal\_add V1x) V3z)))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap (ap c\_2Eextreal\_2Eextreal\_add \\
& V0x) (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap (ap c\_2Eextreal\_2Eextreal\_add \\
& (ap c\_2Eextreal\_2Eextreal\_of\_num c\_2Enum\_2E0)) V0x) = V0x))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty\_2Eextreal\_2Eextreal. (\forall V1x \in ty\_2Eextreal\_2Eextreal. \\
& (\forall V2y \in ty\_2Eextreal\_2Eextreal. ((p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& V0z) (ap (ap c\_2Eextreal\_2Eextreal\_max V1x) V2y))) \Leftrightarrow ((p (ap (ap \\
& c\_2Eextreal\_2Eextreal\_le V0z) V1x)) \vee (p (ap (ap c\_2Eextreal\_2Eextreal\_le \\
& V0z) V2y))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A.27a})})\ (ty\_2Erealx\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V2g \in ( \\
& ty\_2Eextreal\_2Eextreal^{A.27a}).((p\ (ap\ (c\_2Emeasure\_2Emeasure\_space \\
& A.27a)\ V0m)) \wedge ((\forall V3x \in A.27a.((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\
& (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum.2E0))\ (ap\ V1f\ V3x)))) \wedge \\
& (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ (ap\ c\_2Eextreal\_2Eextreal\_of\_num \\
& c\_2Enum.2E0))\ (ap\ V2g\ V3x)))))) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& V1f)\ (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A.27a})\ (2^{(2^{A.27a})}))\ (ap\ (c\_2Emeasure\_2Em\_space \\
& A.27a)\ V0m))\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ V0m)))) \\
& c\_2Emeasure\_2EBorel))) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& V2g)\ (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{A.27a})\ (2^{(2^{A.27a})}))\ (ap\ (c\_2Emeasure\_2Em\_space \\
& A.27a)\ V0m))\ (ap\ (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ V0m)))) \\
& c\_2Emeasure\_2EBorel)))))) \Rightarrow ((ap\ (ap\ (c\_2Elebesgue\_2Epos\_fn\_integral \\
& A.27a)\ V0m)\ (\lambda V4x \in A.27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_add \\
& (ap\ V1f\ V4x))\ (ap\ V2g\ V4x)))) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_add \\
& (ap\ (ap\ (c\_2Elebesgue\_2Epos\_fn\_integral\ A.27a)\ V0m)\ V1f))\ ( \\
& ap\ (ap\ (c\_2Elebesgue\_2Epos\_fn\_integral\ A.27a)\ V0m)\ V2g)))))) \\
& (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in \\
& (2^{(2^{A.27a})}).((ap\ (c\_2Emeasure\_2Espace\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V0x))) \\
& (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{A.27a}).(\forall V1y \in \\
& (2^{(2^{A.27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A.27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& (2^{A.27a})\ (2^{(2^{A.27a})}))\ V0x)\ V1y)) = V1y))) \\
& (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))). \\
& (\forall V1s \in (2^{A-27a}).(\forall V2t \in (2^{A-27a}).(\forall V3u \in \\
& (2^{A-27a}).(((p (ap (c\_2Emeasure\_2Eadditive\ A.27a)\ V0m)) \wedge ((p \\
& (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V2t) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))) \wedge ((p (ap (ap (c\_2Epred\_set\_2EDISJOINT\ A.27a)\ V1s) \\
& V2t)) \wedge (V3u = (ap (ap (c\_2Epred\_set\_2EUNION\ A.27a)\ V1s)\ V2t)))))) \Rightarrow \\
& ((ap (ap (c\_2Emeasure\_2Emeasure\ A.27a)\ V0m)\ V3u) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap (c\_2Emeasure\_2Emeasure\ A.27a)\ V0m)\ V1s) (ap (ap (c\_2Emeasure\_2Emeasure \\
& A.27a)\ V0m)\ V2t))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))). \\
& (((p (ap (c\_2Emeasure\_2Ealgebra\ A.27a) (ap (ap (c\_2Epair\_2E\_2C \\
& (2^{A-27a}) (2^{(2^{A-27a})})) (ap (c\_2Emeasure\_2Em\_space\ A.27a) \\
& V0m)) (ap (c\_2Emeasure\_2Emeasurable\_sets\ A.27a)\ V0m)))) \wedge (( \\
& p (ap (c\_2Emeasure\_2Epositive\ A.27a)\ V0m)) \wedge (p (ap (c\_2Emeasure\_2Ecountably\_additive \\
& A.27a)\ V0m)))) \Rightarrow (p (ap (c\_2Emeasure\_2Eadditive\ A.27a)\ V0m)))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))). \\
& (\forall V1s \in (2^{A-27a}).(\forall V2t \in (2^{A-27a}).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A.27a)\ V0m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V1s) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V2t) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V0m)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a}) (ap (ap (c\_2Epred\_set\_2EINTER \\
& A.27a)\ V1s)\ V2t) (ap (c\_2Emeasure\_2Emeasurable\_sets\ A.27a) \\
& V0m))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0A \in (2^{A-27a}).(\forall V1m \in \\
& (ty\_2Epair\_2Eprod (2^{A-27a}) (ty\_2Epair\_2Eprod (2^{(2^{A-27a})}) \\
& (ty\_2Erealax\_2Ereal^{(2^{A-27a})}))).(((p (ap (c\_2Emeasure\_2Emeasure\_space \\
& A.27a)\ V1m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (2^{A-27a})\ V0A) (ap (c\_2Emeasure\_2Emeasurable\_sets \\
& A.27a)\ V1m)))))) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET\ A.27a)\ V0A) \\
& (ap (c\_2Emeasure\_2Em\_space\ A.27a)\ V1m))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\
& \quad (\forall V1g \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2a \in ( \\
& \quad ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})). (((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& (ty\_2Eextreal\_2Eextreal^{A\_27a})\ V0f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable \\
& \quad A\_27a\ ty\_2Eextreal\_2Eextreal)\ V2a)\ c\_2Emeasure\_2EBorel)))) \wedge \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A\_27a})\ V1g) \\
& \quad (ap\ (ap\ (c\_2Emeasure\_2E measurable\ A\_27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V2a)\ c\_2Emeasure\_2EBorel)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a}) \\
& (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\
& \quad A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ 2) \\
& V3x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_lt\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x)))))) \\
& (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V2a)))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A\_27a)\ V2a))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\
& \quad (\forall V1g \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2a \in ( \\
& \quad ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})). (((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& (ty\_2Eextreal\_2Eextreal^{A\_27a})\ V0f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable \\
& \quad A\_27a\ ty\_2Eextreal\_2Eextreal)\ V2a)\ c\_2Emeasure\_2EBorel)))) \wedge \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A\_27a})\ V1g) \\
& \quad (ap\ (ap\ (c\_2Emeasure\_2E measurable\ A\_27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V2a)\ c\_2Emeasure\_2EBorel)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a}) \\
& (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\
& \quad A\_27a\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ 2) \\
& V3x)\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x)))))) \\
& (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V2a)))\ (ap\ (c\_2Emeasure\_2Esubsets \\
& \quad A\_27a)\ V2a))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A\_27a})\ (2^{(2^{A\_27a})})). (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\
& \quad (\forall V2s \in (2^{A\_27a}). (((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& A\_27a)\ V0a)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A\_27a}) \\
& \quad V1f)\ (ap\ (ap\ (c\_2Emeasure\_2E measurable\ A\_27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V0a)\ c\_2Emeasure\_2EBorel)))) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A\_27a}) \\
& \quad V2s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0a)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& (ty\_2Eextreal\_2Eextreal^{A\_27a})\ (\lambda V3x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul \\
& \quad (ap\ V1f\ V3x))\ (ap\ (ap\ (c\_2Emeasure\_2Eindicator\_fn\ A\_27a)\ V2s) \\
& \quad V3x))))\ (ap\ (ap\ (c\_2Emeasure\_2E measurable\ A\_27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V0a)\ c\_2Emeasure\_2EBorel))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty\_2Epair\_2Eprod \\
& \quad (2^{A.27a})\ (2^{(2^{A.27a})})). (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\
& \quad (\forall V2g \in (ty\_2Eextreal\_2Eextreal^{A.27a}). (((p\ (ap\ (c\_2Emeasure\_2Esigma\_algebra \\
& \quad A.27a)\ V0a)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& \quad V1f)\ (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V0a)\ c\_2Emeasure\_2EBorel)))) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& \quad V2g)\ (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V0a)\ c\_2Emeasure\_2EBorel)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eextreal\_2Eextreal^{A.27a})) \\
& \quad (\lambda V3x \in A.27a. (ap\ (ap\ c\_2Eextreal\_2Eextreal\_max\ (ap\ V1f\ V3x)) \\
& \quad (ap\ V2g\ V3x))))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasurable\ A.27a\ ty\_2Eextreal\_2Eextreal) \\
& \quad V0a)\ c\_2Emeasure\_2EBorel))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2a \in A.27a. (\forall V3b \in \\
& \quad A.27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c\_2Epair\_2E\_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A.27a\ 2)^{A.27b}). (\forall V1v \in \\
& \quad A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\
& \quad A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad A.27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{94}$$



Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}).(\forall V1t \in \\ & (2^{A-27a}).((p\ (ap\ (ap\ (c.2Epred\_set\_2EDISJOINT\ A.27a)\ V0s)\ V1t)) \Leftrightarrow \\ & (\neg(\exists V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V2x)\ V0s)) \wedge \\ & (p\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27a)\ V2x)\ V1t))))))) \end{aligned} \quad (95)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (96)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (97)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (98)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (99)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (100)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg( \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (102)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (103)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (104)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (105)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (106)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (107)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (108)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (109)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (110)$$

### Theorem 1

$$\begin{aligned} & \forall A.27a. \text{nonempty } A.27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A.27a}). \\ & \quad (\forall V1g \in (ty\_2Eextreal\_2Eextreal^{A.27a}). (\forall V2m \in ( \\ & \quad \quad ty\_2Epair\_2Eprod (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) \\ & \quad \quad (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). (\forall V3v \in (ty\_2Epair\_2Eprod \\ & \quad \quad (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\ & \quad ((p (ap (c\_2Emeasure\_2Emeasure\_space A.27a) V2m)) \wedge (p (ap ( \\ & \quad \quad c\_2Emeasure\_2Emeasure\_space A.27a) V3v)) \wedge ((ap (c\_2Emeasure\_2Em\_space \\ & \quad \quad A.27a) V3v) = (ap (c\_2Emeasure\_2Em\_space A.27a) V2m)) \wedge ((ap ( \\ & \quad \quad c\_2Emeasure\_2Emeasurable\_sets A.27a) V3v) = (ap (c\_2Emeasure\_2Emeasurable\_sets \\ & \quad \quad A.27a) V2m)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal^{A.27a}) \\ & \quad \quad V0f) (ap (ap (c\_2ELebesgue\_2ERADON\_F A.27a) V2m) V3v))) \wedge (p (ap ( \\ & \quad \quad (ap (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal^{A.27a}) V1g) (ap ( \\ & \quad \quad ap (c\_2ELebesgue\_2ERADON\_F A.27a) V2m) V3v)))))) \Rightarrow (p (ap (ap \\ & \quad \quad (c\_2Ebool\_2EIN (ty\_2Eextreal\_2Eextreal^{A.27a}) (\lambda V4x \in A.27a. \\ & \quad \quad (ap (ap c\_2Eextreal\_2Eextreal\_max (ap V0f V4x)) (ap V1g V4x))) \\ & \quad \quad (ap (ap (c\_2ELebesgue\_2ERADON\_F A.27a) V2m) V3v)))))) \end{aligned}$$