thm_2Elebesgue_2Emax__fn__seq__mono (TMEp6nCumiD6vUrpsY7GCbN8SDk8a6UwU25)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{2}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 11 *We define* c_2Eextreal_2Eextreal__max *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Ee$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Elebesgue\_2Emax\_\_fn\_\_seq : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a \in (((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum})^{((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum})}) \tag{8}$$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \quad (14)$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V0x)\ V0x))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0g \in ((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V1x \in A\_27a.((ap\ (ap\ (ap\ (c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a)\ V0g)\ \mathsf{c\_2Enum\_2E0})\ V1x) = (ap\ (ap\ V0g\ \mathsf{c\_2Enum\_2E0})\ V1x)))) \wedge (\forall V2g \in ((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V3n \in ty\_2Enum\_2Enum.(\forall V4x \in A\_27a.((ap\ (ap\ (ap\ (c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a)\ V2g)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V4x) = (ap\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_max}\ (ap\ (ap\ (ap\ (c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a)\ V2g)\ V3n)\ V4x))\ (ap\ (ap\ V2g\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))\ V4x))))))) \quad (16)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0g \in ((ty\_2Eextreal\_2Eextreal^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2x \in A\_27a.(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ (ap\ (ap\ (c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a)\ V0g)\ V1n)\ V2x))\ (ap\ (ap\ (ap\ (c\_2Elebesgue\_2Emax\_\_fn\_\_seq\ A\_27a)\ V0g)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2x))))))$$