# thm_2Elebesgue_2Emeasure__space__finite__prod__measure__POW2 (TMHmdf1HzzGzyRt336oQysWmtKi8T9fACsa)

October 26, 2020

**Definition 1** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 4** *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 5** *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 6** *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

**Definition 7** *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

**Definition 8** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

**Definition 9** *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{5}$$

**Definition 10** *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

**Definition 11** *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

**Definition 12** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 13** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

**Definition 14** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{6}$$

Let $c\_2Eextreal\_2ENormal : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \tag{7}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

**Definition 15** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Eextreal\_2EPosInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{12}$$

Let $c\_2Eextreal\_2ENegInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{13}$$

**Definition 16** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

**Definition 17** *We define* c_2Eextreal_2Ereal *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.(ap\ (ap\ (ap$ (c_2Ebool_2

Let $c\_2Eextreal\_2Eextreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{14}$$

**Definition 18** *We define* c_2Eextreal_2Eextreal__of__num *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

Let $c\_2Eextreal\_2Eextreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{15}$$

Let $c\_2Epred\_\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{16}$$

**Definition 19** *We define* c_2Eextreal_2EEXTREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2$

Let $c\_2Eextreal\_2Eextreal\_\_ainv : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_ainv \in (ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal}) \tag{17}$$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{18}$$

**Definition 20** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

**Definition 21** *We define* c_2Eextreal_2Eextreal__lt *to be* $\lambda V0x \in ty\_2Eextreal\_2Eextreal.\lambda V1y \in ty\_2Eext$

**Definition 22** *We define* c_2Emeasure_2Efn__minus *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}$

Let $c\_2Emeasure\_2Em\_\_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{19}$$

**Definition 23** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{20}$$

**Definition 24** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{21}$$

**Definition 25** *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 26** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

**Definition 27** *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

**Definition 28** *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

**Definition 29** *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

**Definition 30** *We define* c_2Epred__set_2EDISJOINT *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

**Definition 31** *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

**Definition 32** *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

**Definition 33** *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Let $c\_2Emeasure\_2Emeasurable\_\_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \tag{22}$$

**Definition 34** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{24}$$

**Definition 35** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{25}$$

**Definition 36** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 37** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

**Definition 38** *We define* c_2Emeasure_2Eindicator__fn *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(\lambda V1x \in A\_27a.(ap$

**Definition 39** *We define* c_2Emeasure_2Epos__simple__fn *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_}$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{27}$$

**Definition 40** *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

**Definition 41** *We define* c_2Elebesgue_2Epsfs *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2I$

Let $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ( (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})} \tag{28}$$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{29}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{30}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{31}$$

**Definition 42** *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

**Definition 43** *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{32}$$

**Definition 44** *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

**Definition 45** *We define* c_2Ereal__sigma_2EREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2$

**Definition 46** *We define* c_2Elebesgue_2Epos__simple__fn__integral *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2E$

**Definition 47** *We define* c_2Elebesgue_2Epsfis *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2$

**Definition 48** *We define* c_2Ereal_2Esup *to be* $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Emin_2E_40 $ty\_2Ereal$

**Definition 49** *We define* c_2Eextreal_2Eextreal__sup *to be* $\lambda V0p \in (2^{ty\_2Eextreal\_2Eextreal}).(ap\ (ap\ (ap$ (c_2E

**Definition 50** *We define* c_2Elebesgue_2Epos__fn__integral *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2$

**Definition 51** *We define* c_2Emeasure_2Efn__plus *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).$

Let $c\_2Eextreal\_2Eextreal\_\_sub : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_sub \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{33}$$

**Definition 52** *We define* c_2Elebesgue_2Eintegral *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

**Definition 53** *We define* c_2Epred__set_2EPREIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V$

**Definition 54** *We define* c_2Elebesgue_2Eprod__measure *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0m0 \in (ty\_2Epair\_2$

**Definition 55** *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2ET$)$.

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{34}$$

**Definition 56** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 57** *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 58** *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{35}$$

**Definition 59** *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

**Definition 60** *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

**Definition 61** *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ ($c_2Ebool_2ECONI

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (36)$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (37)$$

**Definition 62** *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ ($c

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27}$$ (38)

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (39)$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (40)$$

**Definition 63** *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \quad (41)$$

**Definition 64** *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 65** *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

**Definition 66** *We define* c_2Epred__set_2EFUNSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

**Definition 67** *We define* c_2Emeasure_2Ecountably__additive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

**Definition 68** *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

**Definition 69** *We define* c_2Epred__set_2Ecountable *to be* $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ ($c_2Ebool_2E_3F

**Definition 70** *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 71** *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

**Definition 72** *We define* c_2Emeasure_2Esubset__class *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}$

**Definition 73** *We define* c_2Emeasure_2Epositive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Let $c\_2Emeasure\_2Esubsets : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{42}$$

**Definition 74** *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Let $c\_2Emeasure\_2Espace : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \tag{43}$$

**Definition 75** *We define* c_2Emeasure_2Ealgebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

**Definition 76** *We define* c_2Emeasure_2Esigma__algebra *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

**Definition 77** *We define* c_2Emeasure_2Emeasure__space *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A}$

**Definition 78** *We define* c_2Emeasure_2Eadditive *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

**Definition 79** *We define* c_2Epred__set_2ECROSS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in (2^{A}$

**Definition 80** *We define* c_2Epred__set_2EPOW *to be* $\lambda A\_27a : \iota.\lambda V0set \in (2^{A\_27a}).(ap\ (c\_2Epred\_\_set\_2E$

Assume the following.

$$True \tag{44}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{49}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{62}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{63}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{66}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Eextreal\_2Ereal}\ (ap\ c\_2Eextreal\_2ENormal\ V0x)) = V0x)) \tag{67}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2EPosInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2ENegInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2EPosInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ c\_2Eextreal\_2ENegInf) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ c\_2Eextreal\_2ENormal\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x))\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf)))\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2ENegInf)\ (ap\ c\_2Eextreal\_2ENormal\ V1y)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ V1y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ c\_2Eextreal\_2ENormal\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1y))\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf)))\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ c\_2Eextreal\_2EPosInf) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ c\_2Eextreal\_2ENormal\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x))\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2ENegInf)))\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ c\_2Eextreal\_2EPosInf)\ (ap\ c\_2Eextreal\_2ENormal\ V1y)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ V1y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ c\_2Eextreal\_2ENormal\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Eextreal\_2Eextreal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1y))\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2ENegInf)))\wedge((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ (ap\ c\_2Eextreal\_2ENormal\ V1y)) = (ap\ c\_2Eextreal\_2ENormal\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y)))))))))))) \tag{68}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((\neg((ap\ c\_2Eextreal\_2ENormal\ V0x) = c\_2Eextreal\_2ENegInf))\wedge(\neg((ap\ c\_2Eextreal\_2ENormal\ V0x) = c\_2Eextreal\_2EPosInf)))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ c\_2Eextreal\_2ENormal\ (ap\ V0f\ V2x))))\ V1s) = (ap\ c\_2Eextreal\_2ENormal\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V0f)\ V1s)))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m)) \land (((ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)) = (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ V0m)) \land ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m))) \land (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m))) \Rightarrow ((\neg((ap\ V1f\ V2x) = c\_2Eextreal\_2ENegInf)) \land (\neg((ap\ V1f\ V2x) = c\_2Eextreal\_2EPosInf)))))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Eintegral}\ A\_27a)\ V0m)\ V1f) = (ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ V1f\ V3x))\ (ap\ c\_2Eextreal\_2ENormal\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V3x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))))\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)))))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0m0 \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).(\forall V1m1 \in (ty\_2Epair\_2Eprod\ (2^{A\_27b})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27b})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27b})}))).(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m0)) \wedge ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27b)\ V1m1)) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m0))) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27b)\ V1m1))) \wedge (((ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m0)) = (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ V0m0)) \wedge ((ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27b)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27b)\ V1m1)) = (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27b)\ V1m1))))))) \Rightarrow (\forall V2a0 \in (2^{A\_27a}).(\forall V3a1 \in (2^{A\_27b}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2a0)\ (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ V0m0))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27b}))\ V3a1)\ (ap\ ( c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27b)\ V1m1)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Eprod\_\_measure}\ A\_27a\ A\_27b)\ V0m0)\ V1m1)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ V2a0)\ V3a1)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m0)\ V2a0))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27b)\ V1m1)\ V3a1))))))))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V0x))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (2^{A\_27a}).(\forall V1y \in (2^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0x)\ V1y)) = V1y))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1sts)\ V2mu))) = V1sts)))) \tag{75}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(\forall V1sts \in (2^{(2^{A\_27a})}).(\forall V2mu \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).((ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ V0sp)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ V1sts)\ V2mu))) = V2mu)))) \tag{76}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (2^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))\ V0sp)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0sp))))) \tag{77}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0p)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Esubset\_\_class}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0p))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \wedge ((\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1s)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0p))\ V1s))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p))))) \wedge (\forall V2c \in (2^{(2^{A\_27a})}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2Ecountable}\ (2^{A\_27a}))\ V2c)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (2^{A\_27a}))\ V2c)\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V2c))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0p)))))))))) \tag{78}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m)) \Rightarrow ((ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))) \tag{79}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))).\ (\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(\forall V3u \in (2^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1s)\ (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ V0m))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2t)\ (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a)\ V0m))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT}\ A\_27a)\ V1s)\ V2t)) \wedge (V3u = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V1s)\ V2t)))))) \Rightarrow ((ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V3u) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V1s))\ (ap\ (ap\ (c\_2Emeasure\_2Emeasure\ A\_27a)\ V0m)\ V2t)))))))) \tag{80}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})})).(\forall V1m \in (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}).\ (((p\ (ap\ (\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}\ A\_27a)\ V0s)) \wedge ((p\ (ap\ (\ \mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0s))) \wedge ((p\ (ap\ (\mathsf{c\_2Emeasure\_2Epositive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0s))\ V1m)))) \wedge (p\ (ap\ (\mathsf{c\_2Emeasure\_2Eadditive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0s))\ V1m))))))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))\ (ap\ (c\_2Emeasure\_2Espace\ A\_27a)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))\ (ap\ (c\_2Emeasure\_2Esubsets\ A\_27a)\ V0s))\ V1m))))))) \tag{81}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{82}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{83}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{84}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{85}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))\ V2u)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ V2u)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1t)\ V2u)))))))) \tag{86}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT}\ A\_27a)\ V0s)\ V1t)) \Leftrightarrow (\neg(\exists V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))))) \tag{87}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \tag{88}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27b}).(\forall V2x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V2x))\ V0P)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V2x))\ V1Q))))))) \tag{89}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0P \in (2^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ V0P)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)))\wedge((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27c\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27c))\ V0P) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27a))))) \tag{90}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27b}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0P))\wedge(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ V1Q)))\Rightarrow(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ECROSS}\ A\_27a\ A\_27b)\ V0P)\ V1Q)))))) \tag{91}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0set \in (2^{A\_27a}).(\forall V1e \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V1e)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPOW}\ A\_27a)\ V0set)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1e)\ V0set))))) \tag{92}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27b}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EPREIMAGE}\ A\_27a\ A\_27b)\ V0f)\ V1s)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V2x))\ V1s)))))) \tag{93}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{94}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1y)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y)))))) \tag{95}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z))))) \quad (96)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V1s \in (2^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s)) \wedge (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ V0f\ V2x)))))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V0f)\ V1s)))))) \quad (97)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2f\_27 \in (ty\_2Erealax\_2Ereal^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V1f\ V3x))\ (ap\ V2f\_27\ V3x))))\ V0s) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V2f\_27)\ V0s))))))) \quad (98)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2f\_27 \in (ty\_2Erealax\_2Ereal^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \wedge (\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V0s)) \Rightarrow ((ap\ V1f\ V3x) = (ap\ V2f\_27\ V3x))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0s) = (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V2f\_27)\ V0s)))))) \quad (99)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (100)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (101)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (102)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{103}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{104}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg( p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{105}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{106}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{107}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge(( \neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{108}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{109}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{110}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{111}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{112}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{113}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{114}$$

**Theorem 1**

$$\begin{gathered}
\forall A_27a.nonempty\ A_27a\Rightarrow\forall A_27b.nonempty\ A_27b\Rightarrow( \\
\forall V0m0 \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod \\
(2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))).(\forall V1m1 \in \\
(ty_2Epair_2Eprod\ (2^{A_27b})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27b})}) \\
(ty_2Erealax_2Ereal^{(2^{A_27b})}))).(((p\ (ap\ (\mathsf{c_2Emeasure_2Emeasure__space} \\
A_27a)\ V0m0))\wedge((p\ (ap\ (\mathsf{c_2Emeasure_2Emeasure__space}\ A_27b)\ V1m1))\wedge \\
((p\ (ap\ (\mathsf{c_2Epred__set_2EFINITE}\ A_27a)\ (ap\ (c_2Emeasure_2Em__space \\
A_27a)\ V0m0)))\wedge((p\ (ap\ (\mathsf{c_2Epred__set_2EFINITE}\ A_27b)\ (ap\ (c_2Emeasure_2Em__space \\
A_27b)\ V1m1)))\wedge(((ap\ (\mathsf{c_2Epred__set_2EPOW}\ A_27a)\ (ap\ (c_2Emeasure_2Em__space \\
A_27a)\ V0m0)) = (ap\ (c_2Emeasure_2Emeasurable__sets\ A_27a)\ V0m0))\wedge \\
((ap\ (\mathsf{c_2Epred__set_2EPOW}\ A_27b)\ (ap\ (c_2Emeasure_2Em__space \\
A_27b)\ V1m1)) = (ap\ (c_2Emeasure_2Emeasurable__sets\ A_27b)\ V1m1)))))))\Rightarrow \\
(p\ (ap\ (\mathsf{c_2Emeasure_2Emeasure__space}\ (ty_2Epair_2Eprod\ A_27a \\
A_27b))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \\
(ty_2Epair_2Eprod\ (2^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})})\ (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})}))) \\
(ap\ (ap\ (\mathsf{c_2Epred__set_2ECROSS}\ A_27a\ A_27b)\ (ap\ (c_2Emeasure_2Em__space \\
A_27a)\ V0m0))\ (ap\ (c_2Emeasure_2Em__space\ A_27b)\ V1m1)))\ (ap\ ( \\
ap\ (\mathsf{c_2Epair_2E_2C}\ (2^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})})\ (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})})) \\
(ap\ (\mathsf{c_2Epred__set_2EPOW}\ (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap \\
(ap\ (\mathsf{c_2Epred__set_2ECROSS}\ A_27a\ A_27b)\ (ap\ (c_2Emeasure_2Em__space \\
A_27a)\ V0m0))\ (ap\ (c_2Emeasure_2Em__space\ A_27b)\ V1m1))))\ (ap \\
(ap\ (\mathsf{c_2Elebesgue_2Eprod__measure}\ A_27a\ A_27b)\ V0m0)\ V1m1))))))))
\end{gathered}$$