

thm_2Elebesgue_2Epos_fn_integral_add
 (TMXm-
 NmP8QAh3RZ587Vxa9pFQaKiPEqM9tyB)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A. 27b})^{A. 27a}))$

Definition 5 We define `c_2Ecombin_2EC` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A. 27b})^{A. 27a}))$

Definition 6 We define `c_2Ecombin_2EK` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0x \in A. 27a. (\lambda V1y \in A. 27b. V0x))$

Definition 7 We define `c_2Ecombin_2EI` to be $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. 27a (A. 27a^{A. 27a})) A. 27a))$

Definition 8 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A. 27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A. 27a}))))$

Definition 9 We define `c_2Ecombin_2Eo` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0f \in (A. 27b^{A. 27c}). \lambda V1g$

Let `ty_2Enum_2Enum : ι` be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Definition 10 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t)).$

Definition 11 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 12 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_21}))))$

Definition 13 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 18 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (5)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (6)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (7)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))} \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (11)$$

Definition 19 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
 Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (12)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (13)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (14)$$

Definition 20 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 21 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (15)$$

Definition 22 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (16)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (17)$$

Definition 23 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EITSET A_27a A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (18)$$

Definition 24 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal)^{ty_2Erealax_2Ereal} \quad (19)$$

Definition 25 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 26 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal$

Let $c_2Eextreal_2Eextreal_ainv : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_ainv \in (ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal}) \quad (20)$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (21)$$

Definition 27 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 28 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 29 We define $c_2Emeasure_2Efn_minus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a})$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))}) \quad (22)$$

Definition 30 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 31 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (24)$$

Definition 32 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 33 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set$

Definition 34 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 35 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 36 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Let $c_2Erealx_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal)) \quad (25)$$

Definition 37 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Definition 38 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$.

Definition 39 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap \ (c_2Ebool_2E_21 \ 2))$.

Definition 40 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap \ (c_2Ebool_2E_21 \ 2))$.

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \ A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod \ (2^{A_27a}) \ (ty_2Epair_2Eprod \ (2^{(2^{A_27a})}) \ (ty_2Erealx_2Ereal^{(2^{A_27a})})))) \quad (26)$$

Definition 41 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (27)$$

Definition 42 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap \ (ap \ c_2Earithmetic_2E_2B \ V0n))$.

Definition 43 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 44 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap \ (c_2Ebool_2E_21 \ 2)))$.

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) \quad (28)$$

Definition 45 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal)$.

Definition 46 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod \ (2^{A_27a}))$.

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Epair_2ESND \ A_27a \ A_27b \in (A_27b^{(ty_2Epair_2Eprod \ A_27a \ A_27b)}) \quad (29)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Epair_2EFST \ A_27a \ A_27b \in (A_27a^{(ty_2Epair_2Eprod \ A_27a \ A_27b)}) \quad (30)$$

Definition 47 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 48 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 49 We define $c_2Elebesgue_2Epsfis$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 50 We define c_2Ereal_2ESup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40 ty_2Ereal$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (31)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (32)$$

Definition 51 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 52 We define $c_2Elebesgue_2Epos_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a})$

Definition 53 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 54 We define $c_2Eextreal_2Eext_mono_increasing$ to be $\lambda V0f \in (ty_2Eextreal_2Eextreal^{ty_2E$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (\quad (33)$$

$$(2^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})))$$

Definition 55 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 56 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 57 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E.3F$

Definition 58 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))) \quad (34)$$

Definition 59 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 60 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 61 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})$

Definition 62 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a})$

Definition 63 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_2Epred_s$

Definition 64 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap$

Definition 65 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextre$

Definition 66 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V$

Definition 67 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in$

Definition 68 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Eprod$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})(ty_2Epair_2Eprod ty_2Enum_2Enum)) \quad (35)$$

Definition 69 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 70 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (36)$$

Definition 71 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Definition 72 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 73 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECON$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (37)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (38)$$

Definition 74 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)}) \quad (39)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (40)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A-27a})})}) \quad (41)$$

Definition 75 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b})})_{A_27a})_{(A_27a)^{A_27b}}) \end{aligned} \quad (42)$$

Definition 76 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 77 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 78 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 79 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2$

Definition 80 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2$

Definition 81 We define $c_2Emeasure_2Efn_plus$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal^{A_27a}).$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & \quad A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & \quad (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & \quad ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p\ V0P) \wedge (\forall V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (60)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (61)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b\ A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \quad (64)$$

Assume the following.

$$(\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. (((p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ V1y))) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Eextreal_2Eextreal_add\ V0x)\ V1y)))))) \quad (65)$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty_2Eextreal_2Eextreal. (\forall V1x \in ty_2Eextreal_2Eextreal. \\
& \quad (\forall V2y \in ty_2Eextreal_2Eextreal. (\forall V3z \in ty_2Eextreal_2Eextreal. \\
& ((p (ap (ap c_2Eextreal_2Eextreal_le V0w) V1x)) \wedge (p (ap (ap c_2Eextreal_2Eextreal_le \\
& \quad V2y) V3z))) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le (ap (ap c_2Eextreal_2Eextreal_add \\
& \quad V0w) V2y)) (ap (ap c_2Eextreal_2Eextreal_add V1x) V3z))))))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum}). ((p (\\
& \quad ap c_2Eextreal_2Eext_mono_increasing V0f)) \Leftrightarrow (\forall V1n \in \\
& ty_2Enum_2Enum. (p (ap (ap c_2Eextreal_2Eextreal_le (ap V0f V1n)) \\
& \quad (ap V0f (ap c_2Enum_2ESUC V1n))))))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum}). (\forall V1g \in \\
& (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum}). (((\forall V2n \in ty_2Enum_2Enum. \\
& (p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& \quad c_2Enum_2E0)) (ap V0f V2n)))) \wedge ((\forall V3n \in ty_2Enum_2Enum. \\
& (p (ap (ap c_2Eextreal_2Eextreal_le (ap V0f V3n)) (ap V0f (ap c_2Enum_2ESUC \\
& \quad V3n)))) \wedge ((\forall V4n \in ty_2Enum_2Enum. (p (ap (ap c_2Eextreal_2Eextreal_le \\
& \quad (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V1g V4n)))) \wedge \\
& (\forall V5n \in ty_2Enum_2Enum. (p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap V1g V5n)) (ap V1g (ap c_2Enum_2ESUC V5n))))))))) \Rightarrow ((ap c_2Eextreal_2Eextreal_sup \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Eextreal_2Eextreal) \\
& \quad (\lambda V6n \in ty_2Enum_2Enum. (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap V0f V6n)) (ap V1g V6n)))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum))) = \\
& (ap (ap c_2Eextreal_2Eextreal_add (ap c_2Eextreal_2Eextreal_sup \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Eextreal_2Eextreal) \\
& V0f) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) (ap c_2Eextreal_2Eextreal_sup \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Eextreal_2Eextreal) \\
& \quad V1g) (c_2Epred_set_2EUNIV ty_2Enum_2Enum))))))))) \\
& \hspace{15em} (68)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}).(\forall V2g \in (\\
& ty_2Eextreal_2Eextreal^{A_27a}).(\forall V3a \in ty_2Eextreal_2Eextreal. \\
& (\forall V4b \in ty_2Eextreal_2Eextreal.(((p (ap (c_2Emeasure_2Emeasure_space \\
& A_27a) V0m)) \wedge ((p (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal \\
& V3a) (ap (ap (c_2Elebesgue_2Epsfis A_27a) V0m) V1f))) \wedge (p (ap (ap \\
& (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V4b) (ap (ap (c_2Elebesgue_2Epsfis \\
& A_27a) V0m) V2g)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) \\
& (ap (ap c_2Eextreal_2Eextreal_add V3a) V4b)) (ap (ap (c_2Elebesgue_2Epsfis \\
& A_27a) V0m) (\lambda V5x \in A_27a.(ap (ap c_2Eextreal_2Eextreal_add \\
& (ap V1f V5x)) (ap V2g V5x)))))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1r \in ty_2Eextreal_2Eextreal.(\forall V2f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V1r) (ap (ap \\
& (c_2Elebesgue_2Epsfis A_27a) V0m) V2f))) \Leftrightarrow (\exists V3s \in (2^{ty_2Eenum_2Eenum}). \\
& (\exists V4a \in ((2^{A_27a})^{ty_2Eenum_2Eenum}).(\exists V5x \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}). \\
& ((p (ap (ap (ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn A_27a) V0m) \\
& V2f) V3s) V4a) V5x)) \wedge (V1r = (ap (ap (ap (ap (c_2Elebesgue_2Epos_simple_fn_integral \\
& A_27a) V0m) V3s) V4a) V5x)))))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}).(\forall V2s \in (\\
& 2^{ty_2Eenum_2Eenum}).(\forall V3a \in ((2^{A_27a})^{ty_2Eenum_2Eenum}). \\
& (\forall V4x \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).(((p (ap (\\
& c_2Emeasure_2Emeasure_space A_27a) V0m)) \wedge (p (ap (ap (ap (ap (\\
& ap (c_2Emeasure_2Epos_simple_fn A_27a) V0m) V1f) V2s) V3a) V4x)))) \Rightarrow \\
& ((ap (ap (c_2Elebesgue_2Epos_fn_integral A_27a) V0m) V1f) = \\
& (ap (ap (ap (ap (c_2Elebesgue_2Epos_simple_fn_integral A_27a) \\
& V0m) V2s) V3a) V4x)))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_{.27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{.27a}})}) (ty_2Erealax_2Ereal^{(2^{A_{.27a}})}))) . \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}) . (\forall V2g \in (\\
& ty_2Eextreal_2Eextreal^{A_{.27a}}) . ((\forall V3x \in A_{.27a} . ((p (ap (\\
& ap\ c_2Eextreal_2Eextreal_le (ap\ c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap\ V1f\ V3x))) \wedge (p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap\ V1f\ V3x)) (ap\ V2g\ V3x)))))) \Rightarrow (p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap (ap (c_2Elebesgue_2Epos_fn_integral\ A_{.27a})\ V0m)\ V1f)) (\\
& ap (ap (c_2Elebesgue_2Epos_fn_integral\ A_{.27a})\ V0m)\ V2g))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_{.27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{.27a}})}) (ty_2Erealax_2Ereal^{(2^{A_{.27a}})}))) . \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}) . (((p (ap (c_2Emeasure_2Emeasure_space \\
& A_{.27a})\ V0m)) \wedge (\forall V2x \in A_{.27a} . (p (ap (ap\ c_2Eextreal_2Eextreal_le \\
& (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) (ap\ V1f\ V2x)))))) \Rightarrow \\
& (p (ap (ap\ c_2Eextreal_2Eextreal_le (ap\ c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap (ap (c_2Elebesgue_2Epos_fn_integral\ A_{.27a}) \\
& V0m)\ V1f))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}).(\forall V2fi \in \\
& ((ty_2Eextreal_2Eextreal^{A_27a}) ty_2Enum_2Enum).(((p (ap (c_2Emeasure_2Emeasure_space \\
& A_27a) V0m)) \wedge (\forall V3i \in ty_2Enum_2Enum.(p (ap (ap (c_2Ebool_2EIN \\
& (ty_2Eextreal_2Eextreal^{A_27a})) (ap V2fi V3i)) (ap (ap (c_2Emeasure_2Emeasurable \\
& A_27a ty_2Eextreal_2Eextreal) (ap (ap (c_2Epair_2E2C (2^{A_27a}) \\
& (2^{(2^{A_27a})})) (ap (c_2Emeasure_2Em_space A_27a) V0m)) (ap (\\
& c_2Emeasure_2Emeasurable_sets A_27a) V0m)))) c_2Emeasure_2EBorel)))) \wedge \\
& ((\forall V4i \in ty_2Enum_2Enum.(\forall V5x \in A_27a.(p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap (ap V2fi \\
& V4i) V5x)))))) \wedge ((\forall V6x \in A_27a.(p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V1f V6x)))))) \wedge \\
& ((\forall V7x \in A_27a.(p (ap c_2Eextreal_2Eext_mono_increasing \\
& (\lambda V8i \in ty_2Enum_2Enum.(ap (ap V2fi V8i) V7x)))))) \wedge (\forall V9x \in \\
& A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V9x) (ap (c_2Emeasure_2Em_space \\
& A_27a) V0m))) \Rightarrow ((ap c_2Eextreal_2Eextreal_sup (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Enum_2Enum ty_2Eextreal_2Eextreal) (\lambda V10i \in ty_2Enum_2Enum. \\
& (ap (ap V2fi V10i) V9x))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum))) = \\
& (ap (ap V1f V9x)))))) \Rightarrow ((ap (ap (c_2Elebesgue_2Epos_fn_integral \\
& A_27a) V0m) V1f) = (ap c_2Eextreal_2Eextreal_sup (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Enum_2Enum ty_2Eextreal_2Eextreal) (\lambda V11i \in ty_2Enum_2Enum. \\
& (ap (ap (c_2Elebesgue_2Epos_fn_integral A_27a) V0m) (ap V2fi \\
& V11i)))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))))))))
\end{aligned}$$

(74)

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). ((p (ap (c_2Emeasure_2Emeasure_space \\
& \quad A_27a) V0m)) \wedge (p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_27a}) \\
& \quad V1f) (ap (ap (c_2Emeasure_2Emeasurable A_27a ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap (c_2Epair_2E_2C (2^{A_27a}) (2^{(2^{A_27a})})) (ap (c_2Emeasure_2Em_space \\
& \quad A_27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets A_27a) V0m))) \\
& \quad c_2Emeasure_2EBorel))) \Rightarrow ((\exists V2fi \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}). \\
& \quad (\exists V3ri \in (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum}). ((\forall V4x \in \\
& \quad \quad A_27a. (p (ap c_2Eextreal_2Eext_mono_increasing (\lambda V5i \in \\
& \quad \quad ty_2Enum_2Enum. (ap (ap V2fi V5i) V4x)))))) \wedge ((\forall V6x \in A_27a. \\
& \quad ((p (ap (ap (c_2Ebool_2EIN A_27a) V6x) (ap (c_2Emeasure_2Em_space \\
& \quad A_27a) V0m))) \Rightarrow ((ap c_2Eextreal_2Eextreal_sup (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum ty_2Eextreal_2Eextreal) (\lambda V7i \in ty_2Enum_2Enum. \\
& \quad (ap (ap V2fi V7i) V6x))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) = \\
& \quad (ap (ap (c_2Emeasure_2Efn_plus A_27a) V1f) V6x)))) \wedge ((\forall V8i \in \\
& \quad ty_2Enum_2Enum. (p (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) \\
& \quad (ap V3ri V8i)) (ap (ap (c_2ELebesgue_2Epsfis A_27a) V0m) (ap V2fi \\
& \quad V8i)))))) \wedge ((\forall V9i \in ty_2Enum_2Enum. (\forall V10x \in A_27a. \\
& \quad (p (ap (ap c_2Eextreal_2Eextreal_le (ap (ap V2fi V9i) V10x)) (ap \\
& \quad (ap (c_2Emeasure_2Efn_plus A_27a) V1f) V10x)))))) \wedge ((\forall V11i \in \\
& \quad ty_2Enum_2Enum. (\forall V12x \in A_27a. (p (ap (ap c_2Eextreal_2Eextreal_le \\
& \quad (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap (ap V2fi \\
& \quad V11i) V12x)))))) \wedge ((ap (ap (c_2ELebesgue_2Epos_fn_integral \\
& \quad A_27a) V0m) (ap (c_2Emeasure_2Efn_plus A_27a) V1f)) = (ap c_2Eextreal_2Eextreal_sup \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Eextreal_2Eextreal) \\
& \quad (\lambda V13i \in ty_2Enum_2Enum. (ap (ap (c_2ELebesgue_2Epos_fn_integral \\
& \quad A_27a) V0m) (ap V2fi V13i)))))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))))) \wedge \\
& \quad (\exists V14gi \in ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}). \\
& \quad (\exists V15vi \in (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum}). ((\\
& \quad \quad \forall V16x \in A_27a. (p (ap c_2Eextreal_2Eext_mono_increasing \\
& \quad (\lambda V17i \in ty_2Enum_2Enum. (ap (ap V14gi V17i) V16x)))))) \wedge ((\forall V18x \in \\
& \quad A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V18x) (ap (c_2Emeasure_2Em_space \\
& \quad A_27a) V0m))) \Rightarrow ((ap c_2Eextreal_2Eextreal_sup (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum ty_2Eextreal_2Eextreal) (\lambda V19i \in ty_2Enum_2Enum. \\
& \quad (ap (ap V14gi V19i) V18x))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) = \\
& \quad (ap (ap (c_2Emeasure_2Efn_minus A_27a) V1f) V18x)))) \wedge ((\forall V20i \in \\
& \quad ty_2Enum_2Enum. (p (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) \\
& \quad (ap V15vi V20i)) (ap (ap (c_2ELebesgue_2Epsfis A_27a) V0m) (ap V14gi \\
& \quad V20i)))))) \wedge ((\forall V21i \in ty_2Enum_2Enum. (\forall V22x \in A_27a. \\
& \quad (p (ap (ap c_2Eextreal_2Eextreal_le (ap (ap V14gi V21i) V22x)) \\
& \quad (ap (ap (c_2Emeasure_2Efn_minus A_27a) V1f) V22x)))))) \wedge ((\forall V23i \in \\
& \quad ty_2Enum_2Enum. (\forall V24x \in A_27a. (p (ap (ap c_2Eextreal_2Eextreal_le \\
& \quad (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap (ap V14gi \\
& \quad V23i) V24x)))))) \wedge ((ap (ap (c_2ELebesgue_2Epos_fn_integral \\
& \quad A_27a) V0m) (ap (c_2Emeasure_2Efn_minus A_27a) V1f)) = (ap c_2Eextreal_2Eextreal_sup \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Eextreal_2Eextreal) \\
& \quad (\lambda V25i \in ty_2Enum_2Enum. (ap (ap (c_2ELebesgue_2Epos_fn_integral \\
& \quad A_27a) V0m) (ap V14gi V25i)))))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum))))))))) \\
& \quad (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}).((p (ap (c_2Emeasure_2Emeasure_space \\
& A_27a) V0m)) \wedge (\exists V2s \in (2^{ty_2Enum_2Enum}).(\exists V3a \in \\
& ((2^{A_27a})^{ty_2Enum_2Enum}).(\exists V4x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}). \\
& (p (ap (ap (ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn\ A_27a) V0m) \\
V1f) V2s) V3a) V4x)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_27a}) \\
V1f) (ap (ap (c_2Emeasure_2Emeasurable\ A_27a\ ty_2Eextreal_2Eextreal) \\
(ap (ap (c_2Epair_2E_2C (2^{A_27a}) (2^{(2^{A_27a})}) (ap (c_2Emeasure_2Em_space \\
A_27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets\ A_27a) V0m))) \\
c_2Emeasure_2EBorel))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0g \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& ((\forall V1x \in A_27a.(p (ap (ap\ c_2Eextreal_2Eextreal_le (ap \\
c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) (ap\ V0g\ V1x)))) \Rightarrow \\
& ((ap (c_2Emeasure_2Efn_plus\ A_27a) V0g) = V0g)))
\end{aligned} \tag{77}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{78}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{87}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{92}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_{.27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{.27a}})}) (ty_2Erealax_2Ereal^{(2^{A_{.27a}})}))))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). (\forall V2g \in (\\
& ty_2Eextreal_2Eextreal^{A_{.27a}}. (((p (ap (c_2Emeasure_2Emeasure_space \\
& A_{.27a}) V0m)) \wedge (\forall V3x \in A_{.27a}. ((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V1f V3x))) \wedge \\
& (p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap V2g V3x)))))) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_{.27a}})) \\
& V1f) (ap (ap (c_2Emeasure_2Emeasurable A_{.27a} ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Epair_2E_2C (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})) (ap (c_2Emeasure_2Em_space \\
& A_{.27a}) V0m)) (ap (c_2Emeasure_2Emeasurable_sets A_{.27a}) V0m)))) \\
& c_2Emeasure_2EBorel))) \wedge (p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A_{.27a}})) \\
& V2g) (ap (ap (c_2Emeasure_2Emeasurable A_{.27a} ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Epair_2E_2C (2^{A_{.27a}}) (2^{(2^{A_{.27a}})})) (ap (c_2Emeasure_2Em_space \\
& A_{.27a}) V0m)) (ap (c_2Emeasure_2Emeasurable_sets A_{.27a}) V0m)))) \\
& c_2Emeasure_2EBorel)))))) \Rightarrow ((ap (ap (c_2Elebesgue_2Epos_fn_integral \\
& A_{.27a}) V0m) (\lambda V4x \in A_{.27a}. (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap V1f V4x)) (ap V2g V4x)))) = (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap (ap (c_2Elebesgue_2Epos_fn_integral A_{.27a}) V0m) V1f)) (\\
& ap (ap (c_2Elebesgue_2Epos_fn_integral A_{.27a}) V0m) V2g))))))
\end{aligned}$$