

thm_2Elebesgue_2Epos__fn__integral__sum (TM- PRrXP1Kn6wJ9qLAKeKSKtRE2WPVrTTU1J)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 5 We define $c_2Ecombin_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 6 We define $c_2Ecombin_2E_2K$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 7 We define $c_2Ecombin_2E_2S$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 8 We define $c_2Ecombin_2E_2I$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a}) A$

Definition 9 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Definition 10 We define $c_2Ecombin_2E_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \tag{1}$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{2}$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \tag{3}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))})) \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (7)$$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (9)$$

Definition 15 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in 2. (c_2Ebool_2E_21\ 2)\ V0f\ V1s$

Definition 16 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0P)$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 18 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 19 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2EF\ V0s)\ V1t)$

Definition 20 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2EF\ V0s)\ V1t)$

Definition 21 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E7E))$

Let $c_EEnum_EZERO_REP : \iota$ be given. Assume the following.

$$c_EEnum_EZERO_REP \in \omega \tag{10}$$

Let $c_EEnum_EABS_num : \iota$ be given. Assume the following.

$$c_EEnum_EABS_num \in (ty_EEnum_EEnum^{\omega}) \tag{11}$$

Definition 22 We define c_EEnum_E0 to be $(ap c_EEnum_EABS_num c_EEnum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_EEnum_EEnum}) \tag{12}$$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \tag{13}$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \tag{14}$$

Definition 23 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap (c_Emin_E40 (c_Erealax_Ereal_REP_CLASS a)))$

Let $c_Erealax_Etrealm : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal)}) \tag{15}$$

Definition 24 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.(c_Erealax_Ereal_lt T1 T2)$

Definition 25 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.(c_Ereal_Ereal_lte x y)$

Definition 26 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.(c_Ebool_E5C_2F t1 t2)))))$

Definition 27 We define $c_Epred_set_EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_Ebool_E5C_2F s) (c_Epred_set_EINSERT A_27a x))$

Definition 28 We define $c_Epred_set_EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_Ebool_E21 2) (c_Epred_set_EFINITE A_27a s))$

Let $c_Emeasure_Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Emeasure_Emeasurable_sets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_Epair_Eprod\ (2^{A_27a})\ (ty_Epair_Eprod\ (2^{(2^{A_27a})})\ (ty_Erealax_Ereal^{(2^{A_27a})})))}) \tag{16}$$

Let $c_Eextreal_ENormal : \iota$ be given. Assume the following.

$$c_Eextreal_ENormal \in (ty_Eextreal_Eextreal^{ty_Erealax_Ereal}) \tag{17}$$

Definition 29 We define $c_Eextreal_Eextreal_of_num$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ c_Eextreal_Eextreal_of_num)$

Definition 30 We define $c_Earithmetic_EZERO$ to be c_Eenum_ZE0 .

Let $c_Eenum_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_EREP_num \in (\omega^{ty_Eenum_Eenum}) \quad (18)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (\omega^{\omega}) \quad (19)$$

Definition 31 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum.(ap\ c_Eenum_EABS_num)$

Let $c_Earithmetic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2B \in ((ty_Eenum_Eenum)^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (20)$$

Definition 32 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmetic_EBIT1))$

Definition 33 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Eenum_Eenum.V0x$.

Definition 34 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ c_Ebool_ECOND))))$

Definition 35 We define $c_Emeasure_Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap\ c_Emeasure_Eindicator_fn))$

Let $c_Eextreal_Eextreal_mul : \iota$ be given. Assume the following.

$$c_Eextreal_Eextreal_mul \in ((ty_Eextreal_Eextreal)^{ty_Eextreal_Eextreal})^{ty_Eextreal_Eextreal} \quad (21)$$

Let $c_Epred_set_EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_EITSET\ A_27a\ A_27b \in ((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})} \quad (22)$$

Definition 36 We define $c_Eextreal_EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_Eextreal_Eextreal)$

Definition 37 We define $c_Emeasure_Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_Epair_Eprod\ (2^{A_27a}))$

Let $c_Epair_ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_ESND\ A_27a\ A_27b \in (A_27b^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \quad (23)$$

Let $c_Epair_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EFST\ A_27a\ A_27b \in (A_27a^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \quad (24)$$

Definition 38 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 39 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})) (25)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (26)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (27)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} (28)$$

Definition 40 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 41 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (29)$$

Definition 42 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 43 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Definition 44 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 45 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Definition 46 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E.40 ty_2Ereal$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal (30)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal (31)$$

Definition 47 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 48 We define $c_2Elebesgue_2Epos_fn_integral$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{$

Definition 49 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum)}) \quad (32)$$

Definition 50 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 51 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 52 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (33)$$

Definition 53 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Definition 54 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 55 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECONJ$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (34)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (35)$$

Definition 56 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)}) \quad (36)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (37)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (38)$$

Definition 57 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}}) \end{aligned} \quad (39)$$

Definition 58 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 59 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 60 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{$

Definition 61 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 62 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (\\ & (2^{(2^{A_27a})})_{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))} \end{aligned} \quad (40)$$

Definition 63 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 64 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a})$

Definition 65 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E3F$

Definition 66 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})_{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))}) \end{aligned} \quad (41)$$

Definition 67 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 68 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 69 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{$

Definition 70 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{$

Definition 71 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Definition 72 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 73 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 74 We define $c_2Emeasure_2Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1st \in (2^{(2^{A_27a})}).(ap$

Definition 75 We define $c_2Emeasure_2EBorel$ to be $(ap (ap (c_2Emeasure_2Esigma ty_2Eextreal_2Eextreal$

Definition 76 We define $c_2Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V$

Definition 77 We define $c_2Emeasure_2Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Epro$

Definition 78 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (a$

Assume the following.

$$True \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (49)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b\ A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& (((ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A.27a)\ V0f)\ (c_2Epred_set_2EEMPTY \\
& \quad A.27a)) = (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0)) \wedge \\
& (\forall V1e \in A.27a. (\forall V2s \in (2^{A.27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A.27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A.27a) \\
& \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A.27a)\ V1e)\ V2s)) = (ap\ (ap \\
& \quad c_2Eextreal_2Eextreal_add\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A.27a)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A.27a)\ V2s)\ V1e)))))))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& (\forall V1s \in (2^{A.27a}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27a) \\
& \quad V1s)) \wedge (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x) \\
& \quad V1s)) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ c_2Eextreal_2Eextreal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ V0f\ V2x)))))) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A.27a)\ V0f)\ V1s)))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealx_2Ereal^{(2^{A.27a})}))). \\
& ((p\ (ap\ (c_2Emeasure_2Emeasure_space\ A.27a)\ V0m)) \Rightarrow ((ap\ (ap\ (\\
& \quad c_2ELebesgue_2Epos_fn_integral\ A.27a)\ V0m)\ (\lambda V1x \in A.27a. \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))) = (ap\ c_2Eextreal_2Eextreal_of_num \\
& \quad c_2Enum_2E0)))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) (ty_2Epair_2Eprod (2^{(2^{A.27a})}) (ty_2Erealx_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). (\forall V2g \in (\\
& ty_2Eextreal_2Eextreal^{A.27a}). (((p (ap (c_2Emeasure_2Emeasure_space \\
& A.27a) V0m)) \wedge (\forall V3x \in A.27a. ((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V1f V3x))) \wedge \\
& (p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap V2g V3x)))))) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) \\
& V1f) (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Epair_2E.2C (2^{A.27a}) (2^{(2^{A.27a})})) (ap (c_2Emeasure_2Em_space \\
& A.27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets\ A.27a) V0m))) \\
& c_2Emeasure_2EBorel))) \wedge (p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) \\
& V2g) (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Epair_2E.2C (2^{A.27a}) (2^{(2^{A.27a})})) (ap (c_2Emeasure_2Em_space \\
& A.27a) V0m)) (ap (c_2Emeasure_2Emeasurable_sets\ A.27a) V0m))) \\
& c_2Emeasure_2EBorel)))))) \Rightarrow ((ap (ap (c_2Elebesgue_2Epos_fn_integral \\
& A.27a) V0m) (\lambda V4x \in A.27a. (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap V1f V4x)) (ap V2g V4x)))) = (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap (ap (c_2Elebesgue_2Epos_fn_integral\ A.27a) V0m) V1f)) (\\
& ap (ap (c_2Elebesgue_2Epos_fn_integral\ A.27a) V0m) V2g)))))) \\
& (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0a \in (ty_2Epair_2Eprod (2^{A.27a}) (2^{(2^{A.27a})})). (\forall V1f \in \\
& ((ty_2Eextreal_2Eextreal^{A.27a})^{A.27b}). (\forall V2g \in (ty_2Eextreal_2Eextreal^{A.27a}). \\
& (\forall V3s \in (2^{A.27b}). (((p (ap (c_2Epred_set_2EFINITE\ A.27b) \\
& V3s)) \wedge ((p (ap (c_2Emeasure_2Esigma_algebra\ A.27a) V0a)) \wedge ((\\
& \forall V4i \in A.27b. ((p (ap (ap (c_2Ebool_2EIN\ A.27b) V4i) V3s))) \Rightarrow \\
& (p (ap (ap (c_2Ebool_2EIN (ty_2Eextreal_2Eextreal^{A.27a})) (ap \\
& V1f V4i)) (ap (ap (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& V0a) c_2Emeasure_2EBorel)))))) \wedge (\forall V5x \in A.27a. ((p (ap (ap \\
& (c_2Ebool_2EIN\ A.27a) V5x) (ap (c_2Emeasure_2Espace\ A.27a) V0a))) \Rightarrow \\
& ((ap V2g V5x) = (ap (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A.27b) \\
& (\lambda V6i \in A.27b. (ap (ap V1f V6i) V5x))) V3s)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
& (ty_2Eextreal_2Eextreal^{A.27a})) V2g) (ap (ap (c_2Emeasure_2Emeasurable \\
& A.27a\ ty_2Eextreal_2Eextreal) V0a) c_2Emeasure_2EBorel)))))) \\
& (65)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. (\forall V2s \in (2^{A.27a}). ((p (ap (ap (c_2Ebool_2EIN\ A.27a) \\
& V0x) (ap (ap (c_2Epred_set_2EINSERT\ A.27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN\ A.27a) V0x) V2s)))))) \\
& (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ (2^{A_27a}). (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap\ (67) \\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V1s)\ V0x) = V1s)))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((\\ (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\ ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A_27a. ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (68)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \quad (73)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge ((\\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 0p))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee 0p) \vee (p \vee 1q))) \Rightarrow \neg(p \vee 1q))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \vee 0p))) \Rightarrow (p \vee 0p)) \tag{83}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0m \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (ty_2Epair_2Eprod\ (\\
& \quad 2^{(2^{A.27a})})\ (ty_2Erealx_2Ereal^{(2^{A.27a})}))) . (\forall V1f \in \\
& \quad ((ty_2Eextreal_2Eextreal^{A.27a})^{A.27b}) . (\forall V2s \in (2^{A.27b}) . \\
& (((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27b)\ V2s)) \wedge ((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A.27a)\ V0m)) \wedge ((\forall V3i \in A.27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b) \\
& \quad V3i)\ V2s)) \Rightarrow (\forall V4x \in A.27a . (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ V1f \\
& \quad V3i)\ V4x)))))) \wedge (\forall V5i \in A.27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27b)\ V5i)\ V2s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}) \\
& \quad (ap\ V1f\ V5i))\ (ap\ (ap\ (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A.27a})\ (2^{(2^{A.27a})}))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad A.27a)\ V0m))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m))) \\
& \quad c_2Emeasure_2EBorel)))))) \Rightarrow ((ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A.27a)\ V0m)\ (\lambda V6x \in A.27a . (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A.27b)\ (\lambda V7i \in A.27b . (ap\ (ap\ V1f\ V7i)\ V6x)))\ V2s))) = (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMA \\
& \quad A.27b)\ (\lambda V8i \in A.27b . (ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A.27a)\ V0m)\ (ap\ V1f\ V8i))))\ V2s))))))
\end{aligned}$$