

thm_2Elebesgue_2Epos__fn__integral__suminf
(TM-
PVdXhy5AcHz4wzSyAGGAVQyjY9yM5njJz)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define $c_2Ecombin_2EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\lambda$
of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (7)$$

Definition 16 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Epred_set_2EG$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (9)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (10)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (11)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal)^{ty_2Erealax_2Ereal} \quad (12)$$

Definition 18 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (13)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (14)$$

Definition 19 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal)$

Definition 20 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 21 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (15)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (16)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 23 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E40\ ty_2Erealax))$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (18)$$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (19)$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (20)$$

Definition 26 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}).(ap (ap (ap (c_2E$

Definition 27 We define $c_2Eextreal_2Eext_suminf$ to be $\lambda V0f \in (ty_2Eextreal_2Eextreal^{ty_2Enum_2Enum$

Definition 28 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum.2$

Definition 29 We define $c_2Eextreal_2Eext_mono_increasing$ to be $\lambda V0f \in (ty_2Eextreal_2Eextreal^{ty_2E$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Em_space A_27a \in \\ & ((2^{A_27a})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (21)$$

Definition 30 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 31 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 32 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 33 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 34 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2E$

Definition 35 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c$

Definition 36 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ & A_27a \in (((2^{A_27a})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \end{aligned} \quad (22)$$

Definition 37 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (23)$$

Definition 38 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 39 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 40 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) \quad (24)$$

Definition 41 We define $c_Emeasure_Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (25)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (26)$$

Definition 42 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Definition 43 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})} \end{aligned} \quad (27)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \end{aligned} \quad (28)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \end{aligned} \quad (29)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \end{aligned} \quad (30)$$

Definition 44 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 45 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Erealadd : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Erealadd \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \end{aligned} \quad (31)$$

Definition 46 We define $c_2Erealax_2Erealadd$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 47 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2$

Definition 48 We define $c_2E\text{lebesgue_2Epos_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E$

Definition 49 We define $c_2E\text{lebesgue_2Epsfis}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E\text{prod } (2^{A_27a}) (ty_2E$

Definition 50 We define $c_2E\text{lebesgue_2Epos_fn_integral}$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2E\text{pair_2E}E\text{prod } (2^{A_27a})$

Definition 51 We define $c_2E\text{combin_2Eo}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$

Let $c_2E\text{real_2Esum} : \iota$ be given. Assume the following.

$$c_2E\text{real_2Esum} \in ((ty_2E\text{realax_2Ereal}^{(ty_2E\text{realax_2Ereal}^{ty_2E\text{enum_2Eenum}})})_{(ty_2E\text{pair_2E}E\text{prod } ty_2E\text{enum_2Eenum})}) \quad (32)$$

Definition 52 We define $c_2E\text{arithmic_2E_3E}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Definition 53 We define $c_2E\text{arithmic_2E_3E_3D}$ to be $\lambda V0m \in ty_2E\text{enum_2Eenum}.\lambda V1n \in ty_2E\text{enum_2Eenum}$

Let $c_2E\text{realax_2Etreal_neg} : \iota$ be given. Assume the following.

$$c_2E\text{realax_2Etreal_neg} \in ((ty_2E\text{pair_2E}E\text{prod } ty_2E\text{hreal_2Ehreal}^{(ty_2E\text{pair_2E}E\text{prod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})})_{(ty_2E\text{pair_2E}E\text{prod } ty_2E\text{hreal_2Ehreal } ty_2E\text{hreal_2Ehreal})}) \quad (33)$$

Definition 54 We define $c_2E\text{realax_2Ereal_neg}$ to be $\lambda V0T1 \in ty_2E\text{realax_2Ereal}.(ap \ c_2E\text{realax_2Ereal}$

Definition 55 We define $c_2E\text{real_2Ereal_sub}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.\lambda V1y \in ty_2E\text{realax_2Ereal}$

Definition 56 We define $c_2E\text{real_2Eabs}$ to be $\lambda V0x \in ty_2E\text{realax_2Ereal}.(ap \ (ap \ (ap \ (c_2E\text{bool_2ECONJ}$

Let $ty_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2E\text{metric_2Emetric } A0) \quad (34)$$

Let $c_2E\text{metric_2Emetric} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{metric_2Emetric } A_27a \in ((ty_2E\text{metric_2Emetric } A_27a)_{(ty_2E\text{realax_2Ereal}^{(ty_2E\text{pair_2E}E\text{prod } A_27a \ A_27a)})}) \quad (35)$$

Definition 57 We define $c_2E\text{metric_2Emr1}$ to be $(ap \ (c_2E\text{metric_2Emetric } ty_2E\text{realax_2Ereal}) \ (ap \ (c_2E\text{metric_2Emetric } ty_2E\text{realax_2Ereal}))$

Let $c_2E\text{metric_2Edist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{metric_2Edist } A_27a \in ((ty_2E\text{realax_2Ereal}^{(ty_2E\text{pair_2E}E\text{prod } A_27a \ A_27a)})_{(ty_2E\text{pair_2E}E\text{prod } A_27a \ A_27a)}) \quad (36)$$

Let $ty_2E\text{topology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2E\text{topology_2Etopology } A0) \quad (37)$$

Let $c_2E\text{topology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2E\text{topology_2Etopology } A_27a \in ((ty_2E\text{topology_2Etopology } A_27a)_{(2^{(2^{A_27a})})}) \quad (38)$$

Definition 58 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \end{aligned} \quad (39)$$

Definition 59 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 60 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 61 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{$

Definition 62 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 63 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (\\ & (2^{(2^{A_27a})})_{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))}) \end{aligned} \quad (40)$$

Definition 64 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 65 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a})$

Definition 66 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E3F$

Definition 67 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Espace A_27a \in ((2^{A_27a})_{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})}))}) \\ & \end{aligned} \quad (41)$$

Definition 68 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 69 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 70 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{$

Definition 71 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{$

Definition 72 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Definition 73 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal.\lambda V1y \in ty_2Eextreal$

Definition 74 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 75 We define $c_Emeasure_Esigma$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1st \in (2^{(2^{A-27a})}).(ap$

Definition 76 We define $c_Emeasure_EBorel$ to be $(ap (ap (c_Emeasure_Esigma ty_2Eextreal_2Eextreal$

Definition 77 We define $c_Epred_set_2EPREIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V$

Definition 78 We define $c_Emeasure_Emeasurable$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (ty_2Epair_2Epro$

Let $c_Earithmetic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_EEVEN \in (2^{ty_2Enum_2Enum}) \quad (42)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_2Enum_2Enum}) \quad (43)$$

Definition 79 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (44)$$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (45)$$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (46)$$

Definition 80 We define $c_Enumeral_EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 81 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_Earithmetic_E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_Earithmetic_E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_Earithmetic_E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_Earithmetic_E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_Earithmetic_E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_Earithmetic_E_2B V0m) V1n)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_Earithmetic_E_2B V0m) V1n) = (ap (ap c_Earithmetic_E_2B \\ & V1n) V0m)))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)))))) \quad (49)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (50)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (51)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (52)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (53)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (54)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0n))) \quad (55)$$

Assume the following.

$$True \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (60)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (67)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (68)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (69)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))) \quad (70)$$

Assume the following.

$$(\forall V0x \in ty_2Eextreal_2Eextreal.(\forall V1y \in ty_2Eextreal_2Eextreal.(\forall V2z \in ty_2Eextreal_2Eextreal.(((p (ap (ap c_2Eextreal_2Eextreal_le V0x) V1y)) \wedge (p (ap (ap c_2Eextreal_2Eextreal_le V1y) V2z))) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le V0x) V2z)))))) \quad (71)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A.27a}).(\forall V1s \in (2^{A.27a}).(((p (ap (c_2Epred_set_2EFINITE A.27a) V1s)) \wedge (\forall V2x \in A.27a.((p (ap (ap (c_2Ebool_2EIN A.27a) V2x) V1s)) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V0f V2x)))))) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2EEXTREAL_SUM_IMAGE A.27a) V0f) V1s)))))) \quad (72)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). (((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_27a\ V1s)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a\ V2t)) \wedge ((\\
& \quad p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a\ V1s)\ V2t)) \wedge (\forall V3x \in \\
A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a\ V3x)\ V2t)) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ V0f\ V3x)))))) \Rightarrow \\
& (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A_27a\ V0f)\ V1s))\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A_27a\ V0f)\ V2t))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (2^{ty_2Eextreal_2Eextreal}). (\forall V1x \in ty_2Eextreal_2Eextreal. \\
& ((p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ V1x)\ (ap\ c_2Eextreal_2Eextreal_sup \\
& \quad V0p))) \Leftrightarrow (\forall V2y \in ty_2Eextreal_2Eextreal. ((\forall V3z \in \\
ty_2Eextreal_2Eextreal. ((p\ (ap\ V0p\ V3z)) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad V3z)\ V2y)))) \Rightarrow (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le\ V1x)\ V2y))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V2fi \in \\
& ((ty_2Eextreal_2Eextreal^{A_27a})^{ty_2Enum_2Enum}). (((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A_27a\ V0m)) \wedge ((\forall V3i \in ty_2Enum_2Enum. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Eextreal_2Eextreal^{A_27a}))\ (ap\ V2fi\ V3i))\ (ap\ (ap\ (c_2Emeasure_2Emeasurable \\
& \quad A_27a\ ty_2Eextreal_2Eextreal)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a}) \\
& \quad (2^{(2^{A_27a})}))\ (ap\ (c_2Emeasure_2Em_space\ A_27a)\ V0m))\ (ap\ (\\
& \quad c_2Emeasure_2Emeasurable_sets\ A_27a)\ V0m)))\ c_2Emeasure_2EBorel)))) \wedge \\
& ((\forall V4i \in ty_2Enum_2Enum. (\forall V5x \in A_27a. (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ V2fi \\
& \quad V4i)\ V5x)))))) \wedge ((\forall V6x \in A_27a. (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2E0))\ (ap\ V1f\ V6x)))))) \wedge \\
& ((\forall V7x \in A_27a. (p\ (ap\ c_2Eextreal_2Eext_mono_increasing \\
& \quad (\lambda V8i \in ty_2Enum_2Enum. (ap\ (ap\ V2fi\ V8i)\ V7x)))))) \wedge (\forall V9x \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V9x)\ (ap\ (c_2Emeasure_2Em_space \\
& \quad A_27a)\ V0m))) \Rightarrow ((ap\ c_2Eextreal_2Eextreal_sup\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum\ ty_2Eextreal_2Eextreal)\ (\lambda V10i \in ty_2Enum_2Enum. \\
& \quad (ap\ (ap\ V2fi\ V10i)\ V9x)))\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))) = \\
& \quad (ap\ V1f\ V9x)))))) \Rightarrow ((ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A_27a)\ V0m)\ V1f) = (ap\ c_2Eextreal_2Eextreal_sup\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad ty_2Enum_2Enum\ ty_2Eextreal_2Eextreal)\ (\lambda V11i \in ty_2Enum_2Enum. \\
& \quad (ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral\ A_27a)\ V0m)\ (ap\ V2fi \\
& \quad V11i))))\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0m \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (ty_2Epair_2Eprod\ (\\
& \quad 2^{(2^{A.27a})})\ (ty_2Erealx_2Ereal^{(2^{A.27a})}))) . (\forall V1f \in \\
& \quad ((ty_2Eextreal_2Eextreal^{A.27a})^{A.27b}) . (\forall V2s \in (2^{A.27b}) . \\
& \quad (((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27b)\ V2s)) \wedge ((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A.27a)\ V0m)) \wedge ((\forall V3i \in A.27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b) \\
& \quad V3i)\ V2s)) \Rightarrow (\forall V4x \in A.27a . (p\ (ap\ (ap\ c_2Eextreal_2Eextreal_le \\
& \quad (ap\ c_2Eextreal_2Eextreal_of_num\ c_2Enum_2EO))\ (ap\ (ap\ V1f \\
& \quad V3i)\ V4x)))))) \wedge (\forall V5i \in A.27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27b)\ V5i)\ V2s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a}) \\
& \quad (ap\ V1f\ V5i))\ (ap\ (ap\ (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A.27a})\ (2^{(2^{A.27a})}))\ (ap\ (c_2Emeasure_2Em_space \\
& \quad A.27a)\ V0m))\ (ap\ (c_2Emeasure_2Emeasurable_sets\ A.27a)\ V0m))) \\
& \quad c_2Emeasure_2EBorel)))))) \Rightarrow ((ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A.27a)\ V0m)\ (\lambda V6x \in A.27a . (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A.27b)\ (\lambda V7i \in A.27b . (ap\ (ap\ V1f\ V7i)\ V6x)))\ V2s))) = (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAC \\
& \quad A.27b)\ (\lambda V8i \in A.27b . (ap\ (ap\ (c_2Elebesgue_2Epos_fn_integral \\
& \quad A.27a)\ V0m)\ (ap\ V1f\ V8i))))\ V2s)))))) \\
& \hspace{15em} (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0a \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (2^{(2^{A.27a})})) . (\forall V1f \in \\
& \quad ((ty_2Eextreal_2Eextreal^{A.27a})^{A.27b}) . (\forall V2g \in (ty_2Eextreal_2Eextreal^{A.27a}) . \\
& \quad (\forall V3s \in (2^{A.27b}) . (((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27b) \\
& \quad V3s)) \wedge ((p\ (ap\ (c_2Emeasure_2Esigma_algebra\ A.27a)\ V0a)) \wedge ((\\
& \quad \forall V4i \in A.27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ V4i)\ V3s)) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eextreal_2Eextreal^{A.27a})\ (ap \\
& \quad V1f\ V4i))\ (ap\ (ap\ (c_2Emeasure_2Emeasurable\ A.27a\ ty_2Eextreal_2Eextreal) \\
& \quad V0a)\ c_2Emeasure_2EBorel)))))) \wedge (\forall V5x \in A.27a . ((p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A.27a)\ V5x)\ (ap\ (c_2Emeasure_2Espace\ A.27a)\ V0a))) \Rightarrow \\
& \quad ((ap\ V2g\ V5x) = (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A.27b) \\
& \quad (\lambda V6i \in A.27b . (ap\ (ap\ V1f\ V6i)\ V5x)))\ V3s)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Eextreal_2Eextreal^{A.27a})\ V2g)\ (ap\ (ap\ (c_2Emeasure_2Emeasurable \\
& \quad A.27a\ ty_2Eextreal_2Eextreal)\ V0a)\ c_2Emeasure_2EBorel)))))) \\
& \hspace{15em} (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1x \in \\
& A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V0x) (c_2Epred_set_2EUNIV A_27a))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& ((p (ap (ap (c_2Ebool_2EIN A_27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE \\
& A_27a A_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap V2f V3x)) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V1s)))))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V0m) (ap c_2Epred_set_2Ecount \\
& V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (c_2Epred_set_2EFINITE \\
& ty_2Enum_2Enum) (ap c_2Epred_set_2Ecount V0n))))
\end{aligned} \tag{84}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{85}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (86)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (88)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (89)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\ & (2^{A_{27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{27a}})}) (ty_2Erealax_2Ereal^{(2^{A_{27a}})}))). \\ & (\forall V1f \in ((ty_2Eextreal_2Eextreal^{A_{27a}}) ty_2Enum_2Enum). \\ & (((p (ap (c_2Emeasure_2Emeasure_space A_{27a}) V0m)) \wedge ((\forall V2i \in \\ & ty_2Enum_2Enum. (\forall V3x \in A_{27a}. (p (ap (ap c_2Eextreal_2Eextreal_le \\ & (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap (ap V1f \\ & V2i) V3x)))))) \wedge (\forall V4i \in ty_2Enum_2Enum. (p (ap (ap (c_2Ebool_2EIN \\ & (ty_2Eextreal_2Eextreal^{A_{27a}})) (ap V1f V4i)) (ap (ap (c_2Emeasure_2Emeasurable \\ & A_{27a} ty_2Eextreal_2Eextreal) (ap (ap (c_2Epair_2E_2C (2^{A_{27a}}) \\ & (2^{(2^{A_{27a}})})) (ap (c_2Emeasure_2Em_space A_{27a}) V0m)) (ap (\\ & c_2Emeasure_2Emeasurable_sets A_{27a}) V0m)))))) c_2Emeasure_2EBorel)))))) \Rightarrow \\ & ((ap (ap (c_2Elebesgue_2Epos_fn_integral A_{27a}) V0m) (\lambda V5x \in \\ & A_{27a}. (ap c_2Eextreal_2Eext_suminf (\lambda V6i \in ty_2Enum_2Enum. \\ & (ap (ap V1f V6i) V5x)))))) = (ap c_2Eextreal_2Eext_suminf (\lambda V7i \in \\ & ty_2Enum_2Enum. (ap (ap (c_2Elebesgue_2Epos_fn_integral A_{27a}) \\ & V0m) (ap V1f V7i))))))))) \end{aligned}$$