

thm_2Elebesgue_2Epos__simple__fn__add__alt (TMc3SrCboCDg1R3qHqkkkzrxD7Jp6p46PSs)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Definition 4 We define c_2Ebool_2EIN to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27b.nonempty\ A.27b \Rightarrow c_2Epair_2EABS_prod\ A.\lambda 27a.\lambda 27b \in ((ty_2Epair_2Eprod\ A.\lambda 27a.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epair_2EABS_prod (2^{A-27b})^{A-27a}))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27b.nonempty\ A.27b \Rightarrow c_2Epred_set_2EGSPEC\ A.\lambda 27a.\lambda 27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A.\lambda 27a.\lambda 27b)^{(2^{A-27b})^{A-27a}}}) \tag{4}$$

Definition 9 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in ($

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge P \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P \ (\text{ap } (c_2Emin_2E_40$

Definition 12 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (\text{ap } (c_2Epred_set_2EIMAGE$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty \ ty_2Erealax_2Ereal \tag{5}$$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow c_2Emeasure_2Emeasure \ A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})}) (ty_2Epair_2Eprod \ (2^{A_27a}) \ (ty_2Epair_2Eprod \ (2^{(2^{A_27a})}) \ (ty_2Erealax_2Ereal^{(2^{A_27a})})) \tag{6}$$

Definition 13 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in ($

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{7}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{8}$$

Definition 14 We define c_2Enum_2E0 to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})}) (ty_2Epair_2Eprod \ ty_2Enum_2Enum)) \tag{9}$$

Definition 15 We define c_2Ebool_2EF to be $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2. V0t)).$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_3F$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{10}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{11}$$

Definition 17 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (\text{ap } c_2Enum_2EABS_num \ c_2Enum_2ESUC_REP \ V0m).$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (\text{ap } c_2Emin_2E_3D_3D_3E \ V0m \ V1n \ c_2Eprim_rec_2E_3C \ V0m \ V1n).$

Definition 19 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in$

Definition 21 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Erealax)_{ty_2Erealax}) \quad (13)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40) ($

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Etreall)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Erealax)} \quad (16)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 25 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (18)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}\ ty_2Etreall)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 27 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 30 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (20)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (21)$$

Definition 31 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (22)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}}) \end{aligned} \quad (23)$$

Definition 32 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)} \quad (24)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (25)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \\ ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})}})) \end{aligned} \quad (26)$$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b}})))_{A_27a})_{(A_27a)^{A_27b}} \end{aligned} \quad (27)$$

Definition 34 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 35 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ & A_27a \in (((2^{A_27a})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))})) \end{aligned} \quad (28)$$

Definition 36 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 37 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 38 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 39 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).$

Definition 40 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))$

Definition 41 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in \\ & ((2^{A_27a})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))} \end{aligned} \quad (29)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\\ & (2^{(2^{A_27a})})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))} \end{aligned} \quad (30)$$

Definition 42 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Definition 43 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a}).$

Definition 44 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E3F$

Definition 45 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))} \end{aligned} \quad (31)$$

Definition 46 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2E$

Definition 47 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A-27a}).\lambda V1sts \in (2^{(2^{A-27a})})$

Definition 48 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Definition 49 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Definition 50 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A-27a})) (2^{(2^{A-27a})})$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (32)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (33)$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (34)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (35)$$

Definition 51 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap (c_2E$

Definition 52 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E21 (2^{A-27a}))$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (36)$$

Definition 53 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Eextreal_2Eextreal$

Definition 54 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (37)$$

Definition 55 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap (c_2Earithmetic_2E_2B$

Definition 56 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 57 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(\lambda V1x \in A_27a).(ap (c_2E$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})_{ty_2Eextreal_2Eextreal}) \quad (38)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27a})^{(2^{A_27a})})^{((A_27b^{A_27a})^{A_27a})}) \quad (39)$$

Definition 58 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal)$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})_{ty_2Eextreal_2Eextreal}) \quad (40)$$

Definition 59 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a}))$

Assume the following.

$$True \quad (41)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (46)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))) \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (52)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))) \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2a \in ty_2Eextreal_2Eextreal. (\forall V3v2 \in ty_2Erealax_2Ereal. \\ & (\forall V4v5 \in ty_2Erealax_2Ereal. (\forall V5v3 \in ty_2Erealax_2Ereal. \\ & (((ap (ap c_2Eextreal_2Eextreal_add (ap c_2Eextreal_2ENormal \\ & V0x)) (ap c_2Eextreal_2ENormal V1y)) = (ap c_2Eextreal_2ENormal \\ & (ap (ap c_2Erealax_2Ereal_add V0x) V1y))) \wedge (((ap (ap c_2Eextreal_2Eextreal_add \\ & c_2Eextreal_2EPosInf) V2a) = c_2Eextreal_2EPosInf) \wedge (((ap (ap \\ & c_2Eextreal_2Eextreal_add c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf) = \\ & c_2Eextreal_2EPosInf) \wedge (((ap (ap c_2Eextreal_2Eextreal_add \\ & (ap c_2Eextreal_2ENormal V3v2)) c_2Eextreal_2EPosInf) = c_2Eextreal_2EPosInf) \wedge \\ & (((ap (ap c_2Eextreal_2Eextreal_add c_2Eextreal_2ENegInf) \\ & c_2Eextreal_2ENegInf) = c_2Eextreal_2ENegInf) \wedge (((ap (ap c_2Eextreal_2Eextreal_add \\ & c_2Eextreal_2ENegInf) (ap c_2Eextreal_2ENormal V4v5)) = c_2Eextreal_2ENegInf) \wedge \\ & (((ap (ap c_2Eextreal_2Eextreal_add (ap c_2Eextreal_2ENormal \\ & V5v3)) c_2Eextreal_2ENegInf) = c_2Eextreal_2ENegInf))))) \quad (54) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2a \in ty_2Eextreal_2Eextreal. (\forall V3v2 \in ty_2Erealax_2Ereal. \\
& (\forall V4v3 \in ty_2Erealax_2Ereal. (\forall V5v5 \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2ENegInf) \\
& V2a)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V3v2)) c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V4v3)) c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& (ap c_2Eextreal_2ENormal V5v5))) \Leftrightarrow False)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Eextreal_2Eextreal_le (ap \\
& c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) V1y))) \Rightarrow (p (ap \\
& (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Eextreal_2Eextreal_add V0x) V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& (\forall V2z \in ty_2Eextreal_2Eextreal. (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) V1y)) \wedge (p (\\
& ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)) V2z))) \vee ((p (ap (ap c_2Eextreal_2Eextreal_le V1y) \\
& (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0))) \wedge (p (ap (ap \\
& c_2Eextreal_2Eextreal_le V2z) (ap c_2Eextreal_2Eextreal_of_num \\
& c_2Enum_2E0)))))) \Rightarrow ((ap (ap c_2Eextreal_2Eextreal_mul (ap (ap \\
& c_2Eextreal_2Eextreal_add V1y) V2z)) V0x) = (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap (ap c_2Eextreal_2Eextreal_mul V1y) V0x)) (ap (ap c_2Eextreal_2Eextreal_mul \\
& V2z) V0x))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). ((p \text{ (ap} \\
& \text{(c_2Epred_set_2EFINITE } A_{.27a}) V0s)) \Rightarrow (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). \\
& (\forall V2f_{.27} \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). ((\forall V3x \in \\
& A_{.27a}. ((p \text{ (ap (ap (c_2Ebool_2EIN } A_{.27a}) V3x) V0s)) \Rightarrow ((ap V1f V3x) = \\
& (ap V2f_{.27} V3x)))) \Rightarrow ((ap \text{ (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE} \\
& A_{.27a}) V1f) V0s) = (ap \text{ (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE} \\
& A_{.27a}) V2f_{.27}) V0s)))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). ((p \text{ (ap} \\
& \text{(c_2Epred_set_2EFINITE } A_{.27a}) V0s)) \Rightarrow (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). \\
& (\forall V2f_{.27} \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). ((ap \text{ (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE} \\
& A_{.27a}) (\lambda V3x \in A_{.27a}. (ap \text{ (ap c_2Eextreal_2Eextreal_add (ap} \\
& V1f V3x)) (ap V2f_{.27} V3x)))) V0s) = (ap \text{ (ap c_2Eextreal_2Eextreal_add} \\
& (ap \text{ (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE } A_{.27a}) V1f) V0s)) \\
& (ap \text{ (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE } A_{.27a}) V2f_{.27}) V0s)))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p \text{ (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num} \\
& c_2Enum_2E0)) V0x)) \wedge (p \text{ (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num} \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p \text{ (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num} \\
& c_2Enum_2E0)) (ap \text{ (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False}))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{75}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty_2Epair_2Eprod} \\
& (2^{A_{27a}}) (\text{ty_2Epair_2Eprod } (2^{(2^{A_{27a}})}) (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})})))). \\
& (\forall V1f \in (\text{ty_2Eextreal_2Eextreal}^{A_{27a}}). (\forall V2g \in (\\
& \text{ty_2Eextreal_2Eextreal}^{A_{27a}}). (\forall V3s \in (2^{\text{ty_2Enum_2Enum}}). \\
& (\forall V4a \in ((2^{A_{27a}})^{\text{ty_2Enum_2Enum}}). (\forall V5x \in (\text{ty_2Erealax_2Ereal}^{\text{ty_2Enum_2Enum}}). \\
& (\forall V6y \in (\text{ty_2Erealax_2Ereal}^{\text{ty_2Enum_2Enum}}). (((p (ap (\\
& \text{c_2Emeasure_2Emeasure_space } A_{27a}) V0m)) \wedge ((p (ap (ap (ap (ap \\
& (ap (c_2Emeasure_2Epos_simple_fn } A_{27a}) V0m) V1f) V3s) V4a) \\
& V5x)) \wedge (p (ap (ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn } A_{27a}) \\
& V0m) V2g) V3s) V4a) V6y)))))) \Rightarrow (p (ap (ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn \\
& A_{27a}) V0m) (\lambda V7t \in A_{27a}. (ap (ap c_2Eextreal_2Eextreal_add \\
& (ap V1f V7t)) (ap V2g V7t)))))) V3s) V4a) (\lambda V8i \in \text{ty_2Enum_2Enum}. \\
& (ap (ap c_2Erealax_2Ereal_add (ap V5x V8i)) (ap V6y V8i))))))))))
\end{aligned}$$