

thm_2Elebesgue_2Epos__simple__fn__cmul
(TMaojs1LP5vsNBMPgcwPRd1M3vss1WB6n9c)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Definition 4 We define `c_2Ebool_2E_2T` to be $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Epred__set_2EUNIV` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. \text{c_2Ebool_2E_2T}).$

Definition 6 We define `c_2Ebool_2E_2IN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow \ q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \tag{2}$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a \ A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a \ A. 27b))^{((2^{A-27b})^{A-27a})} \tag{3}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 12 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EIMAGE$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal$$
(5)

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in ((ty_2Erealx_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})})$$
(6)

Definition 13 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(7)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(8)

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealx_2Ereal^{(ty_2Erealx_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})$$
(9)

Definition 15 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(10)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(11)

Definition 17 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Definition 18 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 19 We define $c_Earithmic_E_3E$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 20 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_Earithmic_E_3E_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (12)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal_REP_CLASS}) \quad (13)$$

Definition 22 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap\ (c_Emin_E_40\ (t$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal}) \quad (14)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (15)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})} \quad (16)$$

Definition 23 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)$

Definition 24 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal$

Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (17)$$

Definition 25 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Definition 26 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ A_27a A_27b \in & (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b})))})_{A_27a})(A_27a^{A_27b}) \end{aligned} \quad (27)$$

Definition 34 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 35 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets \\ A_27a \in & ((2^{(2^{A_27a})})_{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})})))}) \end{aligned} \quad (28)$$

Definition 36 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 37 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 38 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 39 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 40 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 41 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Em_space A_27a \in \\ & ((2^{A_27a})_{(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealx_2Ereal^{(2^{A_27a})})))}) \end{aligned} \quad (29)$$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Esubsets A_27a \in (\\ & (2^{(2^{A_27a})})_{(ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})))} \end{aligned} \quad (30)$$

Definition 42 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 43 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Definition 44 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E3F$

Definition 45 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (31)$$

Definition 46 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 47 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 48 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 49 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 50 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (32)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (33)$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (34)$$

Definition 51 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 52 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E21\ (2^{A_27a}))$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (35)$$

Definition 53 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Eextreal_2Eextreal_of_num\ V0n))$

Definition 54 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (36)$$

Definition 55 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ V0n))$

Definition 56 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 57 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a).(ap\ (c_2E$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (37)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (38)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (39)$$

Definition 58 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal)$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (40)$$

Definition 59 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^A))$

Let $c_2Erealax_2Etreax_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (41)$$

Definition 60 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Assume the following.

$$True \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2a \in ty_2Eextreal_2Eextreal. (\forall V3v2 \in ty_2Erealax_2Ereal. \\
& (\forall V4v3 \in ty_2Erealax_2Ereal. (\forall V5v5 \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2ENormal \\
& V0x)) (ap c_2Eextreal_2ENormal V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2ENegInf) \\
& V2a)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V3v2)) c_2Eextreal_2EPosInf)) \Leftrightarrow True) \wedge \\
& (((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eextreal_2Eextreal_le \\
& (ap c_2Eextreal_2ENormal V4v3)) c_2Eextreal_2ENegInf)) \Leftrightarrow False) \wedge \\
& ((p (ap (ap c_2Eextreal_2Eextreal_le c_2Eextreal_2EPosInf) \\
& (ap c_2Eextreal_2ENormal V5v5))) \Leftrightarrow False)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (((ap (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2ENegInf) \\
c_2Eextreal_2ENegInf) = c_2Eextreal_2EPosInf) \wedge (((ap (ap c_2Eextreal_2Eextreal_mul \\
& \quad c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf) = c_2Eextreal_2ENegInf) \wedge \\
& \quad ((ap (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2EPosInf) \\
c_2Eextreal_2ENegInf) = c_2Eextreal_2ENegInf) \wedge (((ap (ap c_2Eextreal_2Eextreal_mul \\
& \quad c_2Eextreal_2EPosInf) c_2Eextreal_2EPosInf) = c_2Eextreal_2EPosInf) \wedge \\
& \quad (((ap (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal \\
V0x)) c_2Eextreal_2ENegInf) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V0x)) c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf))) \wedge ((ap \\
& \quad (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2ENegInf) (ap c_2Eextreal_2ENormal \\
V1y)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) (\\
& \quad ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V1y)) c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf))) \wedge ((ap \\
& \quad (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal V0x)) \\
c_2Eextreal_2EPosInf) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V0x)) c_2Eextreal_2EPosInf) c_2Eextreal_2ENegInf))) \wedge ((ap \\
& \quad (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2EPosInf) (ap c_2Eextreal_2ENormal \\
V1y)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) (\\
& \quad ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& \quad (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V1y)) c_2Eextreal_2EPosInf) c_2Eextreal_2ENegInf))) \wedge ((ap (\\
& \quad ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal V0x)) \\
(ap c_2Eextreal_2ENormal V1y)) = (ap c_2Eextreal_2ENormal (ap \\
& \quad (ap c_2Erealax_2Ereal_mul V0x) V1y)))))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& ((p (ap (ap (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& \quad c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Eextreal_2Eextreal_le (ap \\
& \quad \quad c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) V1y))) \Rightarrow (p (ap \\
& \quad (ap c_2Eextreal_2Eextreal_le (ap c_2Eextreal_2Eextreal_of_num \\
& \quad \quad c_2Enum_2E0)) (ap (ap c_2Eextreal_2Eextreal_mul V0x) V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Eextreal_2Eextreal. (\forall V1y \in ty_2Eextreal_2Eextreal. \\
& \quad (\forall V2z \in ty_2Eextreal_2Eextreal. ((ap (ap c_2Eextreal_2Eextreal_mul \\
& V0x) (ap (ap c_2Eextreal_2Eextreal_mul V1y) V2z)) = (ap (ap c_2Eextreal_2Eextreal_mul \\
& \quad (ap (ap c_2Eextreal_2Eextreal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p (ap \\
& \quad (c_2Epred_set_2EFINITE A_27a) V0s)) \Rightarrow (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_27a}). \\
& \quad (\forall V2c \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V2c)) \vee (\forall V3x \in \\
& A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V0s)) \Rightarrow (p (ap (ap c_2Eextreal_2Eextreal_le \\
& \quad (ap c_2Eextreal_2Eextreal_of_num c_2Enum_2E0)) (ap V1f V3x)))))) \Rightarrow \\
& \quad ((ap (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE A_27a) (\lambda V4x \in \\
& \quad A_27a. (ap (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal \\
& \quad \quad V2c)) (ap V1f V4x)))) V0s) = (ap (ap c_2Eextreal_2Eextreal_mul \\
& \quad (ap c_2Eextreal_2ENormal V2c)) (ap (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad \quad A_27a) V1f) V0s)))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& \quad \quad \quad c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (74)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{ty_2Enum_2Enum}). \\ & (\forall V1a \in ((2^{A_27a})^{ty_2Enum_2Enum}). (\forall V2x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\ & (\forall V3m \in (ty_2Epair_2Eprod \ (2^{A_27a}) \ (ty_2Epair_2Eprod \\ & \ (2^{(2^{A_27a})}) \ (ty_2Erealax_2Ereal^{(2^{A_27a})}))))). (\forall V4f \in \\ & (ty_2Eextreal_2Eextreal^{A_27a}). (\forall V5z \in ty_2Erealax_2Ereal. \\ & ((p \ (ap \ (c_2Emeasure_2Emeasure_space \ A_27a) \ V3m)) \wedge ((p \ (ap \ (\\ & \ ap \ (ap \ (ap \ (c_2Emeasure_2Epos_simple_fn \ A_27a) \ V3m) \ V4f) \\ & \ V0s) \ V1a) \ V2x)) \wedge (p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ (ap \ c_2Ereal_2Ereal_of_num \\ & \ c_2Enum_2E0)) \ V5z)))) \Rightarrow (\exists V6s_27 \in (2^{ty_2Enum_2Enum}). \\ & (\exists V7a_27 \in ((2^{A_27a})^{ty_2Enum_2Enum}). (\exists V8x_27 \in \\ & (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (p \ (ap \ (ap \ (ap \ (ap \ (c_2Emeasure_2Epos_simple_fn \\ & \ A_27a) \ V3m) \ (\lambda V9t \in A_27a.(ap \ (ap \ c_2Eextreal_2Eextreal_mul \\ & \ (ap \ c_2Eextreal_2ENormal \ V5z)) \ (ap \ V4f \ V9t)))))) \ V6s_27) \ V7a_27) \\ & \ V8x_27)))))))))) \end{aligned}$$