thm_2Elebesgue_2Epos__simple__fn__integral__add (TMUAYWuqLx-cxg3uzS1RUvksboNqxAKv9g7U)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Eextreal\_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \tag{1}$$

Let $c\_2Eextreal\_2EPosInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2EPosInf \in ty\_2Eextreal\_2Eextreal \tag{2}$$

Let $c\_2Eextreal\_2ENegInf : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENegInf \in ty\_2Eextreal\_2Eextreal \tag{3}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Eextreal\_2Eextreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{4}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{5}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})} \tag{7}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{8}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{9}$$

Definition 7 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{10}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{11}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{12}$$

Definition 9 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 10 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_mul}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{13}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{14}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{15}$$

Definition 11 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (16)$$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (17)$$

Definition 12 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealа$

Let $c\_2Epred\_\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EITSET\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (18)$$

Definition 13 *We define* c_2Ereal__sigma_2EREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2$

Let $c\_2Eextreal\_2ENormal : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (19)$$

Definition 14 *We define* c_2Elebesgue_2Epos__simple__fn__integral *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2E$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 16 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 17 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 18 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (20)$$

Definition 19 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (21)$$

Definition 20 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 21 *We define* $\mathsf{c\_2Epred\_\_set\_2EBIGUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 22 *We define* $\mathsf{c\_2Ecombin\_2Eo}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \quad (22)$$

Definition 23 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (23)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (24)$$

Definition 24 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 25 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 26 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 27 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (25)$$

Definition 28 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_neg}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal}$

Definition 29 *We define* $\mathsf{c\_2Ereal\_2Ereal\_\_sub}$ *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (26)$$

Definition 30 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_lt}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 31 *We define* $\mathsf{c\_2Ereal\_2Ereal\_\_lte}$ *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 32 *We define* $\mathsf{c\_2Ebool\_2ECOND}$ *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 33 *We define* $\mathsf{c\_2Ereal\_2Eabs}$ *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECON}$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{27}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{28}$$

Definition 34 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{29}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{30}$$

Definition 35 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ ($c

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \tag{31}$$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{32}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{33}$$

Definition 36 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{34}$$

Definition 37 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 38 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Let $c\_2Emeasure\_2Emeasurable\_\_sets : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \quad (35)$$

Definition 39 *We define* $\mathsf{c\_2Epred\_\_set\_2EEMPTY}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 40 *We define* $\mathsf{c\_2Epred\_\_set\_2EINTER}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 41 *We define* $\mathsf{c\_2Epred\_\_set\_2EDISJOINT}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 42 *We define* $\mathsf{c\_2Epred\_\_set\_2EFUNSET}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1Q \in ($

Definition 43 *We define* $\mathsf{c\_2Emeasure\_2Ecountably\_\_additive}$ *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod$

Definition 44 *We define* $\mathsf{c\_2Emeasure\_2Epositive}$ *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (t$

Let $c\_2Emeasure\_2Em\_\_space : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \quad (36)$$

Let $c\_2Emeasure\_2Esubsets : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (37)$$

Definition 45 *We define* $\mathsf{c\_2Epred\_\_set\_2ESUBSET}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 46 *We define* $\mathsf{c\_2Epred\_\_set\_2EINJ}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 47 *We define* $\mathsf{c\_2Epred\_\_set\_2Ecountable}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_3F}$

Definition 48 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Let $c\_2Emeasure\_2Espace : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (38)$$

Definition 49 *We define* $\mathsf{c\_2Epred\_\_set\_2EDIFF}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 50 *We define* $\mathsf{c\_2Emeasure\_2Esubset\_\_class}$ *to be* $\lambda A\_27a : \iota.\lambda V0sp \in (2^{A\_27a}).\lambda V1sts \in (2^{(2^{A\_27}}$

Definition 51 *We define* $\mathsf{c\_2Emeasure\_2Ealgebra}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2}$

Definition 52 *We define* $\mathsf{c\_2Emeasure\_2Esigma\_\_algebra}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_2}$

Definition 53 *We define* c_2Emeasure_2Emeasure__space *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\text{-}}$

Definition 54 *We define* c_2Eextreal_2Eextreal__of__num *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

Definition 55 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{39}$$

Definition 56 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 57 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$*.*

Definition 58 *We define* c_2Emeasure_2Eindicator__fn *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(\lambda V1x \in A\_27a.(ap$

Let $c\_2Eextreal\_2Eextreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{40}$$

Definition 59 *We define* c_2Eextreal_2EEXTREAL__SUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Eextreal\_2$

Let $c\_2Eextreal\_2Eextreal\_\_le : \iota$ be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \tag{41}$$

Definition 60 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 61 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 62 *We define* c_2Emeasure_2Epos__simple__fn *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\text{-}}$

Assume the following.

$$True \tag{42}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{47}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{51}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{52}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2a \in ty\_2Eextreal\_2Eextreal.(\forall V3v2 \in ty\_2Erealax\_2Ereal.(\forall V4v5 \in ty\_2Erealax\_2Ereal.(\forall V5v3 \in ty\_2Erealax\_2Ereal.(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ (ap\ c\_2Eextreal\_2ENormal\ V1y)) = (ap\ c\_2Eextreal\_2ENormal\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)))\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ c\_2Eextreal\_2EPosInf)\ V2a) = c\_2Eextreal\_2EPosInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2EPosInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ c\_2Eextreal\_2ENormal\ V3v2))\ c\_2Eextreal\_2EPosInf) = c\_2Eextreal\_2EPosInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ c\_2Eextreal\_2ENegInf)\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf)\wedge(((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ c\_2Eextreal\_2ENegInf)\ (ap\ c\_2Eextreal\_2ENormal\ V4v5)) = c\_2Eextreal\_2ENegInf)\wedge((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ c\_2Eextreal\_2ENormal\ V5v3))\ c\_2Eextreal\_2ENegInf) = c\_2Eextreal\_2ENegInf)))))))))))) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2a \in ty\_2Eextreal\_2Eextreal.(\forall V3v2 \in ty\_2Erealax\_2Ereal.(\forall V4v3 \in ty\_2Erealax\_2Ereal.(\forall V5v5 \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V0x))\ (ap\ c\_2Eextreal\_2ENormal\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y)))\wedge(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2ENegInf)\ V2a))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2EPosInf))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V3v2))\ c\_2Eextreal\_2EPosInf))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ c\_2Eextreal\_2ENegInf))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ c\_2Eextreal\_2ENormal\ V4v3))\ c\_2Eextreal\_2ENegInf))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ c\_2Eextreal\_2EPosInf)\ (ap\ c\_2Eextreal\_2ENormal\ V5v5)))\Leftrightarrow False)))))))))))) \tag{55}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1a\_27 \in ty\_2Erealax\_2Ereal.(((ap\ c\_2Eextreal\_2ENormal\ V0a) = (ap\ c\_2Eextreal\_2ENormal\ V1a\_27))\Leftrightarrow(V0a = V1a\_27)))) \tag{56}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. (((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0x))\wedge(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1y)))\Rightarrow(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ V0x)\ V1y)))))) \tag{57}$$

Assume the following.

$$(\forall V0x \in ty\_2Eextreal\_2Eextreal.(\forall V1y \in ty\_2Eextreal\_2Eextreal. (\forall V2z \in ty\_2Eextreal\_2Eextreal.((((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1y))\wedge(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V2z)))\vee((p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V1y)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))\wedge(p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_le\ V2z)\ (ap\ \mathsf{c\_2Eextreal\_2Eextreal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))))\Rightarrow((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ V1y)\ V2z))\ V0x) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V1y)\ V0x))\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ V2z)\ V0x))))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s))\Rightarrow(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2f\_27 \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).((\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V0s))\Rightarrow((ap\ V1f\ V3x) = (ap\ V2f\_27\ V3x))))\Rightarrow((ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0s) = (ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V2f\_27)\ V0s))))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s))\Rightarrow(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). (\forall V2f\_27 \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ V1f\ V3x))\ (ap\ V2f\_27\ V3x))))\ V0s) = (ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V2f\_27)\ V0s))))))) \tag{60}$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
(2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))). \\
(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2s \in (\\
2^{ty\_2Enum\_2Enum}).(\forall V3a \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}). \\
(\forall V4x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V5g \in \\
(ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V6s\_27 \in (2^{ty\_2Enum\_2Enum}). \\
(\forall V7b \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V8y \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m)) \wedge ((p\ (ap\ (\\
ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Epos\_\_simple\_\_fn}\ A\_27a)\ V0m)\ V1f) \\
V2s)\ V3a)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Epos\_\_simple\_\_fn} \\
A\_27a)\ V0m)\ V5g)\ V6s\_27)\ V7b)\ V8y)))) \Rightarrow (\exists V9z \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
(\exists V10z\_27 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\exists V11c \in \\
((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\exists V12k \in (2^{ty\_2Enum\_2Enum}). \\
((\forall V13t \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V13t)\ (\\
ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m))) \Rightarrow ((ap\ V1f\ V13t) = (ap\ (\\
ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE}\ ty\_2Enum\_2Enum)\ (\lambda V14i \in \\
ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul\ (ap\ c\_2Eextreal\_2ENormal \\
(ap\ V9z\ V14i)))\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Eindicator\_\_fn}\ A\_27a)\ (ap \\
V11c\ V14i))\ V13t))))\ V12k)))) \wedge ((\forall V15t \in A\_27a.((p\ (ap\ (ap \\
(\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V15t)\ (ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a) \\
V0m))) \Rightarrow ((ap\ V5g\ V15t) = (ap\ (ap\ (\mathsf{c\_2Eextreal\_2EEXTREAL\_\_SUM\_\_IMAGE} \\
ty\_2Enum\_2Enum)\ (\lambda V16i \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_mul \\
(ap\ c\_2Eextreal\_2ENormal\ (ap\ V10z\_27\ V16i)))\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Eindicator\_\_fn} \\
A\_27a)\ (ap\ V11c\ V16i))\ V15t))))\ V12k)))) \wedge (((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral} \\
A\_27a)\ V0m)\ V2s)\ V3a)\ V4x) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral} \\
A\_27a)\ V0m)\ V12k)\ V11c)\ V9z)) \wedge (((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral} \\
A\_27a)\ V0m)\ V6s\_27)\ V7b)\ V8y) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral} \\
A\_27a)\ V0m)\ V12k)\ V11c)\ V10z\_27)) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE} \\
ty\_2Enum\_2Enum)\ V12k)) \wedge ((\forall V17i \in ty\_2Enum\_2Enum.((p\ (\\
ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V17i)\ V12k)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \\
(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ V9z\ V17i))))) \wedge \\
((\forall V18i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum) \\
V18i)\ V12k)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
\mathsf{c\_2Enum\_2E0}))\ (ap\ V10z\_27\ V18i))))) \wedge ((\forall V19i \in ty\_2Enum\_2Enum. \\
(\forall V20j \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum) \\
V19i)\ V12k)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V20j) \\
V12k)) \wedge (\neg(V19i = V20j)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT} \\
A\_27a)\ (ap\ V11c\ V19i))\ (ap\ V11c\ V20j))))))) \wedge ((\forall V21i \in ty\_2Enum\_2Enum. \\
((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V21i)\ V12k)) \Rightarrow (p\ (ap \\
(ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ (ap\ V11c\ V21i))\ (ap\ (c\_2Emeasure\_2Emeasurable\_\_sets \\
A\_27a)\ V0m))))) \wedge ((ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ (ap\ (ap \\
(\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Enum\_2Enum\ (2^{A\_27a}))\ V11c)\ V12k)) = \\
(ap\ (c\_2Emeasure\_2Em\_\_space\ A\_27a)\ V0m)))))))))))))))))))))))))
\end{gathered}
\tag{61}
$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1y)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)))))) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z)))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s))\Rightarrow(\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2f\_27 \in (ty\_2Erealax\_2Ereal^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V1f\ V3x))\ (ap\ V2f\_27\ V3x))))\ V0s) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V1f)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_sigma\_2EREAL\_\_SUM\_\_IMAGE}\ A\_27a)\ V2f\_27)\ V0s))))))) \tag{64}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))).(\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V2s \in (2^{ty\_2Enum\_2Enum}).(\forall V3a \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V4x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V5g \in (ty\_2Eextreal\_2Eextreal^{A\_27a}).(\forall V6s\_27 \in (2^{ty\_2Enum\_2Enum}).(\forall V7b \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V8y \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(((p\ (ap\ (\mathsf{c\_2Emeasure\_2Emeasure\_\_space}\ A\_27a)\ V0m))\wedge((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Epos\_\_simple\_\_fn}\ A\_27a)\ V0m)\ V1f)\ V2s)\ V3a)\ V4x))\wedge(p\ (ap\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Epos\_\_simple\_\_fn}\ A\_27a)\ V0m)\ V5g)\ V6s\_27)\ V7b)\ V8y))))\Rightarrow(\exists V9s\_27\_27 \in (2^{ty\_2Enum\_2Enum}).(\exists V10c \in ((2^{A\_27a})^{ty\_2Enum\_2Enum}).(\exists V11z \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Emeasure\_2Epos\_\_simple\_\_fn}\ A\_27a)\ V0m)\ (\lambda V12x \in A\_27a.(ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ V1f\ V12x))\ (ap\ V5g\ V12x))))\ V9s\_27\_27)\ V10c)\ V11z))\wedge((ap\ (ap\ c\_2Eextreal\_2Eextreal\_\_add\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral}\ A\_27a)\ V0m)\ V2s)\ V3a)\ V4x))\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral}\ A\_27a)\ V0m)\ V6s\_27)\ V7b)\ V8y)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral}\ A\_27a)\ V0m)\ V9s\_27\_27)\ V10c)\ V11z))))))))))))))))$$