

# thm\_2Elebesgue\_2Epos\_\_simple\_\_fn\_\_integral\_\_sum (TMNxp2SmFz6Xx7fuK2e814xpJNSwTAvjcee)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))$

**Definition 4** We define  $c\_2Ecombin\_2E\_C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 5** We define  $c\_2Ecombin\_2E\_K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2E\_S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 7** We define  $c\_2Ecombin\_2E\_I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_S A\_27a (A\_27a^{A\_27a}) A\_27a$

**Definition 8** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 9** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})$

**Definition 10** We define  $c\_2Ecombin\_2E\_o$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Emeasure\_2Emeasure : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasure\ A\_27a \in (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})}))\ (ty\_2Erealx\_2Ereal^{(2^{A\_27a})})) \tag{3}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (4)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (5)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (6)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (8)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $c\_2Erealax\_2Etrealmul\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ . Let  $c\_2Epred\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EITSET \\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (14)$$

**Definition 16** We define  $c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal)$ . Let  $ty\_2Eextreal\_2Eextreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eextreal\_2Eextreal \quad (15)$$

Let  $c\_2Eextreal\_2ENormal : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2ENormal \in (ty\_2Eextreal\_2Eextreal^{ty\_2Erealax\_2Ereal}) \quad (16)$$

**Definition 17** We define  $c\_2Elebesgue\_2Epos\_simple\_fn\_integral$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Epair\_2Ebool\_2EIN)$ . Let  $c\_2Eextreal\_2Eextreal\_add : \iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_add \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (17)$$

**Definition 18** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EIN)$ .

**Definition 19** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(\text{ap } V1f\ V0x)))$

**Definition 20** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 21** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \end{aligned} \quad (18)$$

**Definition 22** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(\text{ap } (c\_2Ebool\_2E21\ 2)\ (\lambda V2z \in 2))))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (19)$$

**Definition 23** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b})$

**Definition 24** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(\text{ap } (c\_2Epred\_set\_2EGSPEC\ P))$





Let  $c\_2Emeasure\_2Emeasurable\_sets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Emeasurable\_sets\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \quad (33)$$

**Definition 43** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 44** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EAND)\ s\ t)$

**Definition 45** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EOR)\ s\ t)$

**Definition 46** We define  $c\_2Epred\_set\_2EFUNSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A\_27a}). \lambda V1Q \in (2^{A\_27b}). (ap\ (c\_2Ebool\_2EAND)\ P\ Q)$

**Definition 47** We define  $c\_2Emeasure\_2Ecountably\_additive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 48** We define  $c\_2Emeasure\_2Epositive$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})}))$

Let  $c\_2Emeasure\_2Em\_space : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Em\_space\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ (2^{(2^{A\_27a})})\ (ty\_2Erealax\_2Ereal^{(2^{A\_27a})})))}) \quad (34)$$

Let  $c\_2Emeasure\_2Esubsets : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Esubsets\ A\_27a \in (2^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (35)$$

**Definition 49** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EAND)\ s\ t)$

**Definition 50** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EAND)\ f\ s)$

**Definition 51** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EAND)\ s\ (c\_2Ebool\_2E3F))$

**Definition 52** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EOR)\ s\ t)$

Let  $c\_2Emeasure\_2Espace : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emeasure\_2Espace\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))}) \quad (36)$$

**Definition 53** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2EAND)\ s\ t)$

**Definition 54** We define  $c\_2Emeasure\_2Esubset\_class$  to be  $\lambda A\_27a : \iota. \lambda V0sp \in (2^{A\_27a}). \lambda V1sts \in (2^{(2^{A\_27a})})$

**Definition 55** We define  $c\_2Emeasure\_2Ealgebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 56** We define  $c\_2Emeasure\_2Esigma\_algebra$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (2^{(2^{A\_27a})}))$

**Definition 57** We define `c_2Emeasure_2Emeasure_space` to be  $\lambda A_{.27a} : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^A$

**Definition 58** We define `c_2Eextreal_2Eextreal_of_num` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Eextreal$

**Definition 59** We define `c_2Earithmetic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Earithmetic_2E_2B` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (37)$$

**Definition 60** We define `c_2Earithmetic_2EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 61** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 62** We define `c_2Emeasure_2Eindicator_fn` to be  $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A-27a}).(\lambda V1x \in A_{.27a}.(ap$

Let `c_2Eextreal_2Eextreal_mul` :  $\iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_mul \in ((ty\_2Eextreal\_2Eextreal^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (38)$$

**Definition 63** We define `c_2Eextreal_2EEXTREAL_SUM_IMAGE` to be  $\lambda A_{.27a} : \iota.\lambda V0f \in (ty\_2Eextreal$

Let `c_2Eextreal_2Eextreal_le` :  $\iota$  be given. Assume the following.

$$c\_2Eextreal\_2Eextreal\_le \in ((2^{ty\_2Eextreal\_2Eextreal})^{ty\_2Eextreal\_2Eextreal}) \quad (39)$$

**Definition 64** We define `c_2Epred_set_2EINSERT` to be  $\lambda A_{.27a} : \iota.\lambda V0x \in A_{.27a}.\lambda V1s \in (2^{A-27a}).(ap\ (c$

**Definition 65** We define `c_2Epred_set_2EFINITE` to be  $\lambda A_{.27a} : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c\_2Ebool\_2E21\ (2$

**Definition 66** We define `c_2Emeasure_2Epos_simple_fn` to be  $\lambda A_{.27a} : \iota.\lambda V0m \in (ty\_2Epair\_2Eprod (2^A$

Let `c_2Earithmetic_2EEVEN` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (40)$$

Let `c_2Earithmetic_2EODD` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (41)$$

**Definition 67** We define `c_2Eprim_rec_2EPRE` to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E21$

Let `c_2Earithmetic_2EEXP` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (42)$$

Let `c_2Earithmetic_2E_2D` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (43)$$

Let `c_2Earithmetic_2E_2A` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (44)$$

**Definition 68** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 69** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2 V0n) V0n)$ .

**Definition 70** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)$ .

**Definition 71** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1x \in A\_27a.(ap (ap c\_2Epred\_set\_2EDELETE V0s) V1x)$ .

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\ & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\ & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\ & V0m) V1n)))))))))) \quad (46) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\ & ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (47) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (48) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap c\_2Enum\_2ESUC V1n)) V0m)))) \quad (49) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0n))) \quad (50)$$

Assume the following.

$$True \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (54)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (56)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (57)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (62)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge ((p\ V2C) \vee (p\ V0A))) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (66)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (67)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (68)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (69)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v)))) \quad (70)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (71)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}). (((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\ & A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\ & A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap\ (ap\ c\_2Eextreal\_2Eextreal\_mul \\ & (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0))\ V0x) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num \\ & c\_2Enum\_2E0))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal. (p\ (ap\ (ap\ c\_2Eextreal\_2Eextreal\_le \\ & V0x)\ V0x))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Eextreal\_2Eextreal. ((ap\ (ap\ c\_2Eextreal\_2Eextreal\_add \\ & V0x)\ (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) = V0x)) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\ & (((ap\ (ap\ (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE\ A\_27a)\ V0f)\ (c\_2Epred\_set\_2EEMPTY \\ & A\_27a)) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1e \in A\_27a. (\forall V2s \in (2^{A\_27a}). ((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & A\_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE\ A\_27a) \\ & V0f)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1e)\ V2s)) = (ap\ (ap \\ & c\_2Eextreal\_2Eextreal\_add\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE \\ & A\_27a)\ V0f)\ (ap\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V2s)\ V1e)))))))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eextreal\_2Eextreal^{A\_27a}). \\ & (\forall V1s \in (2^{A\_27a}). (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\ & V1s)) \wedge (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x) \\ & V1s)) \Rightarrow ((ap\ V0f\ V2x) = (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)))))) \Rightarrow \\ & ((ap\ (ap\ (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE\ A\_27a)\ V0f)\ V1s) = \\ & (ap\ c\_2Eextreal\_2Eextreal\_of\_num\ c\_2Enum\_2E0)))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V2g \in ( \\
& ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V3s \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V4a \in ((2^{A.27a})^{ty\_2Enum\_2Enum}).(\forall V5x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (\forall V6y \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(((p (ap ( \\
& c.2Emeasure\_2Emeasure\_space\ A.27a)\ V0m)) \wedge ((p (ap (ap (ap \\
& (ap (c.2Emeasure\_2Epos\_simple\_fn\ A.27a)\ V0m)\ V1f)\ V3s)\ V4a) \\
& V5x)) \wedge (p (ap (ap (ap (ap (ap (c.2Emeasure\_2Epos\_simple\_fn\ A.27a) \\
& V0m)\ V2g)\ V3s)\ V4a)\ V6y)))))) \Rightarrow (p (ap (ap (ap (ap (ap (c.2Emeasure\_2Epos\_simple\_fn \\
& A.27a)\ V0m)\ (\lambda V7t \in A.27a.(ap (ap\ c.2Eextreal\_2Eextreal\_add \\
& (ap\ V1f\ V7t))\ (ap\ V2g\ V7t))))))\ V3s)\ V4a)\ (\lambda V8i \in ty\_2Enum\_2Enum. \\
& (ap (ap\ c.2Erealax\_2Ereal\_add\ (ap\ V5x\ V8i))\ (ap\ V6y\ V8i))))))))))))) \\
& (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty\_2Epair\_2Eprod \\
& (2^{A.27a}) (ty\_2Epair\_2Eprod (2^{(2^{A.27a})}) (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V2s \in ( \\
& 2^{ty\_2Enum\_2Enum}).(\forall V3a \in ((2^{A.27a})^{ty\_2Enum\_2Enum}). \\
& (\forall V4x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V5g \in \\
& (ty\_2Eextreal\_2Eextreal^{A.27a}).(\forall V6y \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (((p (ap (c.2Emeasure\_2Emeasure\_space\ A.27a)\ V0m)) \wedge ((p (ap ( \\
& ap (ap (ap (ap (c.2Emeasure\_2Epos\_simple\_fn\ A.27a)\ V0m)\ V1f) \\
& V2s)\ V3a)\ V4x)) \wedge (p (ap (ap (ap (ap (ap (c.2Emeasure\_2Epos\_simple\_fn \\
& A.27a)\ V0m)\ V5g)\ V2s)\ V3a)\ V6y)))))) \Rightarrow ((ap (ap\ c.2Eextreal\_2Eextreal\_add \\
& (ap (ap (ap (ap (c.2Elebesgue\_2Epos\_simple\_fn\_integral\ A.27a) \\
& V0m)\ V2s)\ V3a)\ V4x))\ (ap (ap (ap (ap (c.2Elebesgue\_2Epos\_simple\_fn\_integral \\
& A.27a)\ V0m)\ V2s)\ V3a)\ V6y)) = (ap (ap (ap (ap (c.2Elebesgue\_2Epos\_simple\_fn\_integral \\
& A.27a)\ V0m)\ V2s)\ V3a)\ (\lambda V7i \in ty\_2Enum\_2Enum.(ap (ap\ c.2Erealax\_2Ereal\_add \\
& (ap\ V4x\ V7i))\ (ap\ V6y\ V7i))))))))))))) \\
& (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D c\_2Earithmic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))))))))) \\
& \tag{81}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg (p (ap (ap \\
& (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EEMPTY A\_27a)))))) \\
& \tag{82}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (\forall V2s \in (2^{A\_27a}). ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\
& V0x) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V2s)))))) \\
& \tag{83}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1s \in \\
& (2^{A\_27a}). ((\neg (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) V1s))) \Leftrightarrow ((ap \\
& (ap (c\_2Epred\_set\_2EDELETE A\_27a) V1s) V0x) = V1s)))) \\
& \tag{84}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (ap (ap (c\_2Epred\_set\_2EINSERT \\
& A\_27a) V1y) (c\_2Epred\_set\_2EEMPTY A\_27a)))) \Leftrightarrow (V0x = V1y)))) \\
& \tag{85}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{27a}})}). (( \\
& (p (ap V0P (c\_2Epred\_set\_2EEMPTY A_{27a}))) \wedge (\forall V1s \in (2^{A_{27a}}). \\
& (((p (ap (c\_2Epred\_set\_2EFINITE A_{27a}) V1s)) \wedge (p (ap V0P V1s))) \Rightarrow \\
& (\forall V2e \in A_{27a}. ((\neg(p (ap (ap (c\_2Ebool\_2EIN A_{27a}) V2e) V1s))) \Rightarrow \\
& (p (ap V0P (ap (ap (c\_2Epred\_set\_2EINSERT A_{27a}) V2e) V1s)))))) \Rightarrow \\
& (\forall V3s \in (2^{A_{27a}}). ((p (ap (c\_2Epred\_set\_2EFINITE A_{27a}) \\
& V3s)) \Rightarrow (p (ap V0P V3s))))))
\end{aligned} \tag{86}$$

Assume the following.

$$(\forall V0x \in \text{ty\_2Erealax\_2Ereal}. ((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x)) \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in \text{ty\_2Enum\_2Enum}. (\forall V1m \in \text{ty\_2Enum\_2Enum}. ( \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Ereal\_2Ereal\_of\_num V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num V0n)) (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1m))) \Leftrightarrow \text{True}) \wedge (((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1m)))) \Leftrightarrow ((V0n = c\_2Enum\_2E0) \wedge (V1m = c\_2Enum\_2E0))) \wedge ((p (ap (ap \\
& c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1m)))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D V1m) V0n))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (\text{ty\_2Erealax\_2Ereal}^{A_{27a}}). \\
& (((ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE A_{27a}) V0f) (c\_2Epred\_set\_2EEMPTY \\
& A_{27a})) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \wedge (\forall V1e \in \\
& A_{27a}. (\forall V2s \in (2^{A_{27a}}). ((p (ap (c\_2Epred\_set\_2EFINITE \\
& A_{27a}) V2s)) \Rightarrow ((ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE A_{27a}) \\
& V0f) (ap (ap (c\_2Epred\_set\_2EINSERT A_{27a}) V1e) V2s)) = (ap (ap \\
& c\_2Erealax\_2Ereal\_add (ap V0f V1e)) (ap (ap (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\
& A_{27a}) V0f) (ap (ap (c\_2Epred\_set\_2EDELETE A_{27a}) V2s) V1e))))))
\end{aligned} \tag{89}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{90}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \tag{91}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (92)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (93)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (94)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (95)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (99)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (100)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (101)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (102)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (103)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (104)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow ( \\ & \forall V0m \in (ty\_2Epair\_2Eprod \ (2^{A.27a}) \ (ty\_2Epair\_2Eprod \ ( \\ & \quad 2^{(2^{A.27a})}) \ (ty\_2Erealax\_2Ereal^{(2^{A.27a})}))) . (\forall V1f \in \\ & \quad ((ty\_2Eextreal\_2Eextreal^{A.27a})^{A.27b}) . (\forall V2s \in (2^{ty\_2Enum\_2Enum}) . \\ & (\forall V3a \in ((2^{A.27a})^{ty\_2Enum\_2Enum}) . (\forall V4x \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{A.27b}) . \\ & \quad (\forall V5P \in (2^{A.27b}) . (((p \ (ap \ (c\_2Emeasure\_2Emeasure\_space \\ & \quad A.27a) \ V0m)) \wedge ((\forall V6i \in A.27b. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A.27b) \\ & \quad V6i) \ V5P))) \Rightarrow (p \ (ap \ (ap \ (ap \ (ap \ (ap \ (c\_2Emeasure\_2Epos\_simple\_fn \\ & \quad A.27a) \ V0m) \ (ap \ V1f \ V6i)) \ V2s) \ V3a) \ (ap \ V4x \ V6i)))))) \wedge ((p \ (ap \ (c\_2Epred\_set\_2EFINITE \\ & \quad A.27b) \ V5P)) \wedge (\neg(V5P = (c\_2Epred\_set\_2EEMPTY \ A.27b)))))) \Rightarrow (( \\ & \quad p \ (ap \ (ap \ (ap \ (ap \ (ap \ (c\_2Emeasure\_2Epos\_simple\_fn \ A.27a) \ V0m) \\ & \quad (\lambda V7t \in A.27a. (ap \ (ap \ (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE \\ & \quad A.27b) \ (\lambda V8i \in A.27b. (ap \ (ap \ V1f \ V8i) \ V7t))) \ V5P))) \ V2s) \ V3a) \ ( \\ & \quad \lambda V9i \in ty\_2Enum\_2Enum. (ap \ (ap \ (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\ & \quad A.27b) \ (\lambda V10j \in A.27b. (ap \ (ap \ V4x \ V10j) \ V9i))) \ V5P)))) \wedge ((ap \ ( \\ & \quad ap \ (ap \ (ap \ (c\_2Elebesgue\_2Epos\_simple\_fn\_integral \ A.27a) \\ & \quad V0m) \ V2s) \ V3a) \ (\lambda V11j \in ty\_2Enum\_2Enum. (ap \ (ap \ (c\_2Ereal\_sigma\_2EREAL\_SUM\_IMAGE \\ & \quad A.27b) \ (\lambda V12i \in A.27b. (ap \ (ap \ V4x \ V12i) \ V11j))) \ V5P))) = (ap \ (ap \\ & \quad (c\_2Eextreal\_2EEXTREAL\_SUM\_IMAGE \ A.27b) \ (\lambda V13i \in A.27b. \\ & \quad (ap \ (ap \ (ap \ (ap \ (c\_2Elebesgue\_2Epos\_simple\_fn\_integral \ A.27a) \\ & \quad V0m) \ V2s) \ V3a) \ (ap \ V4x \ V13i)))) \ V5P))))))))) \end{aligned}$$