

thm_2Elebesgue_2Epsfis__cmul (TMM- sNc7zjDLbSPC6Q76HpKY74AADwaJgrmP)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealx_2Ereal^{(2^{A_27a})})))) \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Definition 5 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (5)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (6)$$

Definition 9 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in ($

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_s$

Definition 13 We define $c_2Ebool_2E_F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_F)$.

Definition 15 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Definition 16 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (11)$$

Definition 19 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (t$
Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (12)$$

Definition 20 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 21 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ebool_2E.5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E.21 2) (\lambda V2t \in 2$

Definition 23 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E.21 2$

Definition 24 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E.21 2$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (13)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty ty_2Eextreal_2Eextreal \quad (14)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (15)$$

Definition 25 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Eextreal_2ENormal$

Definition 26 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (17)$$

Definition 27 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 28 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 29 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 30 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 31 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (19)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (20)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \quad (21)$$

Definition 32 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal)$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (22)$$

Definition 33 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a}))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (23)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (24)$$

Definition 34 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Definition 35 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a}))\ (ty_2Eprod\ (2^{A_27a}))$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a}))}\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})}))\ (ty_2Erealax_2Ereal^{(2^{A_27a})})) \end{aligned} \quad (25)$$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (26)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (27)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (28)$$

Definition 36 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 37 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (29)$$

Definition 38 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 39 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A.27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal)$

Definition 40 We define $c_2Elebesgue_2Epos_simple_fn_integral$ to be $\lambda A.27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)$

Definition 41 We define $c_2Elebesgue_2Epsfis$ to be $\lambda A.27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A.27a})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal))$

Definition 42 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal)^{(ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})})(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum) \quad (30)$$

Definition 43 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 44 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 45 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (31)$$

Definition 46 We define $c_2Erealx_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.(ap\ c_2Erealx_2Ereal$

Definition 47 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 48 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECON$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (32)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (33)$$

Definition 49 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealx_2Ereal)\ (ap\ (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (34)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (35)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (36)$$

Definition 50 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b})}))^{A_27a})^{(A_27a^{A_27b})}) \quad (37)$$

Definition 51 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 52 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty$

Definition 53 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 54 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 55 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in ((2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (38)$$

Definition 56 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E3F$

Definition 57 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}$

Definition 58 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E3F$

Definition 59 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})})})) \quad (39)$$

Definition 60 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 61 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a}$

Definition 62 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a}$

Definition 63 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a}$

Definition 64 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a}$

Assume the following.

$$True \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (44)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealx_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}).(\forall V2s \in (\\
& 2^{ty_2Enum_2Enum}).(\forall V3a \in ((2^{A.27a})ty_2Enum_2Enum)). \\
& (\forall V4x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(\forall V5z \in \\
& ty_2Erealx_2Ereal.(((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& A.27a)\ V0m)) \wedge ((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Emeasure_2Epos_simple_fn \\
& A.27a)\ V0m)\ V1f)\ V2s)\ V3a)\ V4x)) \wedge (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V5z)))))) \Rightarrow ((p\ (ap\ (\\
& ap\ (ap\ (ap\ (ap\ (c_2Emeasure_2Epos_simple_fn\ A.27a)\ V0m)\ (\lambda V6x \in \\
& A.27a.(ap\ (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal \\
& V5z))\ (ap\ V1f\ V6x))))))\ V2s)\ V3a)\ (\lambda V7i \in ty_2Enum_2Enum.(ap\ (ap \\
& c_2Erealx_2Ereal_mul\ V5z)\ (ap\ V4x\ V7i)))))) \wedge ((ap\ (ap\ (ap\ (ap\ (\\
& c_2ELebesgue_2Epos_simple_fn_integral\ A.27a)\ V0m)\ V2s)\ V3a) \\
& (\lambda V8i \in ty_2Enum_2Enum.(ap\ (ap\ c_2Erealx_2Ereal_mul\ V5z) \\
& (ap\ V4x\ V8i)))))) = (ap\ (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal \\
& V5z))\ (ap\ (ap\ (ap\ (ap\ (c_2ELebesgue_2Epos_simple_fn_integral \\
& A.27a)\ V0m)\ V2s)\ V3a)\ V4x)))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& A.27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& A.27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).(\exists V1q \in A.27a. \\
& (\exists V2r \in A.27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b) \\
& V1q)\ V2r))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p))))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \tag{73}$$

Theorem 1
$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0m \in (\text{ty_2Epair_2Eprod} \\ & (2^{A_{27a}}) (\text{ty_2Epair_2Eprod } (2^{(2^{A_{27a}})}) (\text{ty_2Erealax_2Ereal}^{(2^{A_{27a}})})))). \\ & (\forall V1f \in (\text{ty_2Eextreal_2Eextreal}^{A_{27a}}). (\forall V2a \in \text{ty_2Eextreal_2Eextreal}. \\ & (\forall V3z \in \text{ty_2Erealax_2Ereal}. (((p (ap (c_2Emeasure_2Emeasure_space \\ & A_{27a}) V0m)) \wedge ((p (ap (ap (c_2Ebool_2EIN \text{ty_2Eextreal_2Eextreal} \\ & V2a) (ap (ap (c_2Elebesgue_2Epsfis A_{27a}) V0m) V1f))) \wedge (p (ap (ap \\ & c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\ & V3z)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \text{ty_2Eextreal_2Eextreal}) (ap \\ & (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal V3z)) \\ & V2a)) (ap (ap (c_2Elebesgue_2Epsfis A_{27a}) V0m) (\lambda V4x \in A_{27a}. \\ & (ap (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal \\ & V3z)) (ap V1f V4x)))))))))) \end{aligned}$$