

thm_2Elebesgue_2Epsfis__intro (TMTy-
PhqLesLUwWHsjdQFnMb4wRLidb9Y8Jg)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).f (V1g))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p (ap\ P\ x))$) of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (ty_2Erealax_2Ereal_mul) (c_2Erealax_2Ereal_mul)))$.
Let $c_2Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (5)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))} \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$.

Definition 11 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $c_2Erealax_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (8)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty ty_2Eextreal_2Eextreal \quad (9)$$

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (10)$$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (14)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (16)$$

Definition 14 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \quad (17)$$

Definition 15 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (18)$$

Definition 16 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2Eextreal_of_num\ n)$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (20)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 21 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0f \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 22 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 23 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 24 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 26 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasure A_27a \in ((ty_2Erealax_2Ereal^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})) (22)$$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod (2^{A_27a}) (ty_2Epair_2Eprod (2^{(2^{A_27a})}) (ty_2Erealax_2Ereal^{(2^{A_27a})})))) (23)$$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) (24)$$

Definition 27 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})ty_2Eextreal_2Eextreal) (25)$$

Definition 29 We define $c_2Eextreal_2EEEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2$

Definition 30 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) (26)$$

Definition 31 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) (27)$$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (32)$$

Definition 46 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 47 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 48 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (33)$$

Definition 49 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 50 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 51 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (34)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (35)$$

Definition 52 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (36)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (37)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A_27a)}})) \quad (38)$$

Definition 53 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b})}))_{A_27a})_{(A_27a)^{A_27b}}) \end{aligned} \quad (39)$$

Definition 54 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 55 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 56 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b})$

Definition 57 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 58 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (ty_2$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (\\ & (2^{(2^{A_27a})})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \end{aligned} \quad (40)$$

Definition 59 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 60 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a})$

Definition 61 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap$

Definition 62 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})_{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \end{aligned} \quad (41)$$

Definition 63 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 64 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota.\lambda V0sp \in (2^{A_27a}).\lambda V1sts \in (2^{(2^{A_27a})})$

Definition 65 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})$

Definition 66 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})$

Definition 67 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod (2^{A_27a}) (2^{(2^{A_27a})})$

Assume the following.

$$True \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\
& \quad (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\
& \quad V2x))\ (ap\ V0f\ V3y))))))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\
& (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\
& \quad A_27a)\ V0x) = V0x)) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\
& \quad A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2ENegInf) \\
c_2Eextreal_2ENegInf) = c_2Eextreal_2EPosInf) \wedge ((ap (ap c_2Eextreal_2Eextreal_mul \\
& c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf) = c_2Eextreal_2ENegInf) \wedge \\
& ((ap (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2EPosInf) \\
c_2Eextreal_2ENegInf) = c_2Eextreal_2ENegInf) \wedge ((ap (ap c_2Eextreal_2Eextreal_mul \\
& c_2Eextreal_2EPosInf) c_2Eextreal_2EPosInf) = c_2Eextreal_2EPosInf) \wedge \\
& ((ap (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal \\
V0x)) c_2Eextreal_2ENegInf) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V0x)) c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf))) \wedge ((ap \\
& (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2ENegInf) (ap c_2Eextreal_2ENormal \\
V1y)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) (\\
& ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V1y)) c_2Eextreal_2ENegInf) c_2Eextreal_2EPosInf))) \wedge ((ap \\
& (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal V0x)) \\
c_2Eextreal_2EPosInf) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V0x)) c_2Eextreal_2EPosInf) c_2Eextreal_2ENegInf))) \wedge ((ap \\
& (ap c_2Eextreal_2Eextreal_mul c_2Eextreal_2EPosInf) (ap c_2Eextreal_2ENormal \\
V1y)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) (\\
& ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_2Ereal_of_num \\
c_2Enum_2E0))) (ap c_2Eextreal_2ENormal (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Eextreal_2Eextreal) \\
& (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
V1y)) c_2Eextreal_2EPosInf) c_2Eextreal_2ENegInf))) \wedge ((ap (\\
& ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal V0x)) \\
(ap c_2Eextreal_2ENormal V1y)) = (ap c_2Eextreal_2ENormal (ap \\
& (ap c_2Erealax_2Ereal_mul V0x) V1y)))))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A.27a}). \\
& \quad (\forall V1s \in (2^{A.27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A.27a) \\
& \quad V1s)) \Rightarrow ((ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A.27a)\ (\lambda V2x \in \\
& \quad A.27a.(ap\ c_2Eextreal_2ENormal\ (ap\ V0f\ V2x))))\ V1s) = (ap\ c_2Eextreal_2ENormal \\
& \quad (ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A.27a)\ V0f)\ V1s)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& \quad (\forall V1A \in (2^{A.27a}). (((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A.27a)\ V0m)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A.27a})\ V1A)\ (ap\ (c_2Emeasure_2Emeasurable_sets \\
& \quad A.27a)\ V0m)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal) \\
& \quad (ap\ c_2Eextreal_2ENormal\ (ap\ (ap\ (c_2Emeasure_2Emeasure\ A.27a) \\
& \quad V0m)\ V1A)))\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A.27a)\ V0m)\ (ap\ (c_2Emeasure_2Eindicator_fn \\
& \quad A.27a)\ V1A)))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& \quad (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}). (\forall V2a \in ty_2Eextreal_2Eextreal. \\
& \quad (\forall V3z \in ty_2Erealax_2Ereal. (((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A.27a)\ V0m)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal) \\
& \quad V2a)\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A.27a)\ V0m)\ V1f))) \wedge (p\ (ap\ (ap \\
& \quad c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
& \quad V3z)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal)\ (ap \\
& \quad (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal\ V3z)) \\
& \quad V2a))\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A.27a)\ V0m)\ (\lambda V4x \in A.27a. \\
& \quad (ap\ (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal \\
& \quad V3z))\ (ap\ V1f\ V4x)))))))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (\\
& \quad 2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) . (\forall V1f \in \\
& ((ty_2Eextreal_2Eextreal^{A_27a})^{A_27b}) . (\forall V2a \in (ty_2Eextreal_2Eextreal^{A_27b}) . \\
& \quad (\forall V3P \in (2^{A_27b}) . (((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A_27a)\ V0m)) \wedge ((\forall V4i \in A_27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& \quad V4i)\ V3P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal) \\
& \quad (ap\ V2a\ V4i))\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A_27a)\ V0m)\ (ap\ V1f\ V4i)))))) \wedge \\
& \quad (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V3P)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Eextreal_2Eextreal)\ (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& \quad A_27b)\ V2a)\ V3P))\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A_27a)\ V0m)\ (\lambda V5t \in \\
& \quad A_27a . (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A_27b)\ (\lambda V6i \in \\
& \quad A_27b . (ap\ (ap\ V1f\ V6i)\ V5t)))\ V3P))))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0t \in 2 . ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2 . ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2 . (\forall V1B \in 2 . (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2 . (\forall V1B \in 2 . (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0A \in 2 . (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2 . (\forall V1q \in 2 . (\forall V2r \in 2 . (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2 . (\forall V1q \in 2 . (\forall V2r \in 2 . (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow ((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p \ V0p) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ 2) \ V1q) \ V2r) \ V3s))) \Leftrightarrow \\
& (((p \ V0p) \vee ((p \ V1q) \vee \neg(p \ V3s))) \wedge ((p \ V0p) \vee ((\neg(p \ V2r)) \vee \neg(p \ V1q)))) \wedge \\
& (((p \ V0p) \vee ((\neg(p \ V2r)) \vee \neg(p \ V3s))) \wedge ((\neg(p \ V1q) \vee ((p \ V2r) \vee \neg(\\
& p \ V0p)))) \wedge ((p \ V1q) \vee ((p \ V3s) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V0p))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{76}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (\\
& \quad 2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})}))) . (\forall V1a \in \\
& \quad ((2^{A_27a})^{A_27b}) . (\forall V2x \in (ty_2Erealax_2Ereal^{A_27b}) . \\
& \quad (\forall V3P \in (2^{A_27b}) . (((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& \quad A_27a)\ V0m)) \wedge ((\forall V4i \in A_27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& \quad V4i)\ V3P)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ (ap\ V1a\ V4i))\ (ap \\
& \quad (c_2Emeasure_2Emeasurable_sets\ A_27a)\ V0m)))))) \wedge ((\forall V5i \in \\
& \quad A_27b . ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V5i)\ V3P)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ V2x\ V5i)))))) \wedge \\
& \quad (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V3P)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Eextreal_2Eextreal)\ (ap\ c_2Eextreal_2ENormal\ (ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE \\
& \quad A_27b)\ (\lambda V6i \in A_27b . (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ V2x \\
& \quad V6i))\ (ap\ (ap\ (c_2Emeasure_2Emeasure\ A_27a)\ V0m)\ (ap\ V1a\ V6i)))))) \\
& \quad V3P)))\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A_27a)\ V0m)\ (\lambda V7t \in A_27a . \\
& \quad (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ A_27b)\ (\lambda V8i \in \\
& \quad A_27b . (ap\ (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal \\
& \quad (ap\ V2x\ V8i))\ (ap\ (ap\ (c_2Emeasure_2Eindicator_fn\ A_27a)\ (ap \\
& \quad V1a\ V8i))\ V7t))))))\ V3P)))))))))
\end{aligned}$$