

thm_2Elebesgue_2Epsfis__present
(TMHwSWX4diFXDKQv6t27v99XtBjM4i9hiyn)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Emeasure_2Em_space : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Em_space\ A_27a \in ((2^{A_27a})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealx_2Ereal^{(2^{A_27a})})))) \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Definition 5 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (5)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (6)$$

Definition 9 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in ($

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set$

Definition 13 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 14 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c$

Definition 15 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (11)$$

Definition 18 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (t$
Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (12)$$

Definition 19 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 20 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 21 We define $c_2Ebool_2E.5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E.21\ 2)\ (\lambda V2t \in 2$

Definition 22 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E.21\ 2)$

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E.21\ 2)$

Let $c_2Emeasure_2Emeasurable_sets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasurable_sets\ A_27a \in ((2^{(2^{A_27a})})(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})))) \quad (13)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (14)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (15)$$

Definition 24 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Eextreal_2ENormal\ (ap\ c_2Ebool_2E.21\ 2)$

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (17)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC_REP\ (ap\ c_2Enum_2EABS_num\ (ap\ c_2Ebool_2E.21\ 2)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Ebool_2E.21\ 2)$

Definition 28 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 30 We define $c_2Emeasure_2Eindicator_fn$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1x \in A_27a.(ap$

Let $c_2Eextreal_2Eextreal_mul : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_mul \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (19)$$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (20)$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \quad (21)$$

Definition 31 We define $c_2Eextreal_2EEXTREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Eextreal_2Eextreal$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal} \quad (22)$$

Definition 32 We define $c_2Emeasure_2Epos_simple_fn$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (23)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (24)$$

Definition 33 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a$

Definition 34 We define $c_2Elebesgue_2Epsfs$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2E$

Let $c_2Emeasure_2Emeasure : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Emeasure\ A_27a \in (\\ (ty_2Erealax_2Ereal^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A_27a})})\ (ty_2Erealax_2Ereal^{(2^{A_27a})})} \end{aligned} \quad (25)$$

Definition 46 We define $c_2Erealx_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.(ap\ c_2Erealx_2Ereal$

Definition 47 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 48 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECON$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (32)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (33)$$

Definition 49 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealx_2Ereal)\ (ap\ (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (34)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (35)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (36)$$

Definition 50 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b})}))^{A_27a})^{(A_27a)^{A_27b}}) \quad (37)$$

Definition 51 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 52 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 53 We define $c_2Epred_set_2EFUNSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in ($

Definition 54 We define $c_2Emeasure_2Ecountably_additive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod$

Definition 55 We define $c_2Emeasure_2Epositive$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty$

Let $c_2Emeasure_2Esubsets : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Esubsets\ A_27a \in (2^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (38)$$

Definition 56 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 57 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27b}$

Definition 58 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 59 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Let $c_2Emeasure_2Espace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emeasure_2Espace\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))}) \quad (39)$$

Definition 60 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E3F$

Definition 61 We define $c_2Emeasure_2Esubset_class$ to be $\lambda A_27a : \iota. \lambda V0sp \in (2^{A_27a}). \lambda V1sts \in (2^{(2^{A_27a})})$

Definition 62 We define $c_2Emeasure_2Ealgebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 63 We define $c_2Emeasure_2Esigma_algebra$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Definition 64 We define $c_2Emeasure_2Emeasure_space$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (2^{(2^{A_27a})}))$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in \\ 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow & (45) \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A_{.27a}}) (ty_2Epair_2Eprod (2^{(2^{A_{.27a}})}) (ty_2Erealax_2Ereal^{(2^{A_{.27a}})}))) \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A_{.27a}}). (\forall V2s \in (\\
& 2^{ty_2Enum_2Enum}). (\forall V3a \in ((2^{A_{.27a}}) ty_2Enum_2Enum). \\
& (\forall V4x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V5g \in \\
& (ty_2Eextreal_2Eextreal^{A_{.27a}}). (\forall V6s_{.27} \in (2^{ty_2Enum_2Enum}). \\
& (\forall V7b \in ((2^{A_{.27a}}) ty_2Enum_2Enum). (\forall V8y \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& (((p (ap (c_2Emeasure_2Emeasure_space A_{.27a}) V0m)) \wedge (p (ap (\\
& ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn A_{.27a}) V0m) V1f) \\
& V2s) V3a) V4x)) \wedge (p (ap (ap (ap (ap (ap (c_2Emeasure_2Epos_simple_fn \\
& A_{.27a}) V0m) V5g) V6s_{.27}) V7b) V8y)))))) \Rightarrow (\exists V9z \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& (\exists V10z_{.27} \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\exists V11c \in \\
& ((2^{A_{.27a}}) ty_2Enum_2Enum). (\exists V12k \in (2^{ty_2Enum_2Enum}). \\
& ((\forall V13t \in A_{.27a}. ((p (ap (ap (c_2Ebool_2EIN A_{.27a}) V13t) (\\
& ap (c_2Emeasure_2Em_space A_{.27a}) V0m))) \Rightarrow ((ap V1f V13t) = (ap (\\
& ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE ty_2Enum_2Enum) (\lambda V14i \in \\
& ty_2Enum_2Enum. (ap (ap c_2Eextreal_2Eextreal_mul (ap c_2Eextreal_2ENormal \\
& (ap V9z V14i))) (ap (ap (c_2Emeasure_2Eindicator_fn A_{.27a}) (ap \\
& V11c V14i)) V13t)))))) \wedge ((\forall V15t \in A_{.27a}. ((p (ap (ap \\
& (c_2Ebool_2EIN A_{.27a}) V15t) (ap (c_2Emeasure_2Em_space A_{.27a}) \\
& V0m))) \Rightarrow ((ap V5g V15t) = (ap (ap (c_2Eextreal_2EEXTREAL_SUM_IMAGE \\
& ty_2Enum_2Enum) (\lambda V16i \in ty_2Enum_2Enum. (ap (ap c_2Eextreal_2Eextreal_mul \\
& (ap c_2Eextreal_2ENormal (ap V10z_{.27} V16i))) (ap (ap (c_2Emeasure_2Eindicator_fn \\
& A_{.27a}) (ap V11c V16i)) V15t)))))) V12k)))) \wedge (((ap (ap (ap (ap (c_2ELebesgue_2Epos_simple_fn_integral \\
& A_{.27a}) V0m) V2s) V3a) V4x) = (ap (ap (ap (ap (c_2ELebesgue_2Epos_simple_fn_integral \\
& A_{.27a}) V0m) V12k) V11c) V9z)) \wedge (((ap (ap (ap (ap (c_2ELebesgue_2Epos_simple_fn_integral \\
& A_{.27a}) V0m) V6s_{.27}) V7b) V8y) = (ap (ap (ap (ap (c_2ELebesgue_2Epos_simple_fn_integral \\
& A_{.27a}) V0m) V12k) V11c) V10z_{.27})) \wedge (p (ap (c_2Epred_set_2EFINITE \\
& ty_2Enum_2Enum) V12k)) \wedge ((\forall V17i \in ty_2Enum_2Enum. ((p (\\
& ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V17i) V12k)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap V9z V17i)))))) \wedge \\
& ((\forall V18i \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) \\
& V18i) V12k)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap V10z_{.27} V18i)))))) \wedge ((\forall V19i \in ty_2Enum_2Enum. \\
& (\forall V20j \in ty_2Enum_2Enum. ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) \\
& V19i) V12k)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V20j) \\
& V12k)) \wedge (\neg (V19i = V20j)))))) \Rightarrow (p (ap (ap (c_2Epred_set_2EDISJOINT \\
& A_{.27a}) (ap V11c V19i)) (ap V11c V20j)))))) \wedge ((\forall V21i \in ty_2Enum_2Enum. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V21i) V12k)) \Rightarrow (p (ap \\
& (ap (c_2Ebool_2EIN (2^{A_{.27a}}) (ap V11c V21i)) (ap (c_2Emeasure_2Emeasurable_sets \\
& A_{.27a}) V0m)))))) \wedge ((ap (c_2Epred_set_2EBIGUNION A_{.27a}) (ap (ap \\
& (c_2Epred_set_2EIMAGE ty_2Enum_2Enum (2^{A_{.27a}}) V11c) V12k)) = \\
& (ap (c_2Emeasure_2Em_space A_{.27a}) V0m)))))))))
\end{aligned}$$

(46)

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\ & \quad (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ \\ & \quad V1q)\ V2r)))))) \\ & \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & \quad A_27b\ A_27c)\ V0f) (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y)) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \\ & \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v) (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y) (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \\ & \end{aligned} \tag{51}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0m \in (ty_2Epair_2Eprod \\
& (2^{A.27a})\ (ty_2Epair_2Eprod\ (2^{(2^{A.27a})})\ (ty_2Erealax_2Ereal^{(2^{A.27a})}))). \\
& (\forall V1f \in (ty_2Eextreal_2Eextreal^{A.27a}).(\forall V2g \in (\\
& ty_2Eextreal_2Eextreal^{A.27a}).(\forall V3a \in ty_2Eextreal_2Eextreal. \\
& (\forall V4b \in ty_2Eextreal_2Eextreal.(((p\ (ap\ (c_2Emeasure_2Emeasure_space \\
& A.27a)\ V0m)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal) \\
& V3a)\ (ap\ (ap\ (c_2Elebesgue_2Epsfis\ A.27a)\ V0m)\ V1f)))) \wedge (p\ (ap\ (ap \\
& (c_2Ebool_2EIN\ ty_2Eextreal_2Eextreal)\ V4b)\ (ap\ (ap\ (c_2Elebesgue_2Epsfis \\
& A.27a)\ V0m)\ V2g)))))) \Rightarrow (\exists V5z \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}). \\
& (\exists V6z.27 \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).(\exists V7c \in \\
& (2^{A.27a})^{ty_2Eenum_2Eenum}).(\exists V8k \in (2^{ty_2Eenum_2Eenum}). \\
& ((\forall V9t \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V9t)\ (ap \\
& (c_2Emeasure_2Em_space\ A.27a)\ V0m))) \Rightarrow ((ap\ V1f\ V9t) = (ap\ (ap\ (\\
& c_2Eextreal_2EEXTREAL_SUM_IMAGE\ ty_2Eenum_2Eenum)\ (\lambda V10i \in \\
& ty_2Eenum_2Eenum.(ap\ (ap\ c_2Eextreal_2Eextreal_mul\ (ap\ c_2Eextreal_2ENormal \\
& (ap\ V5z\ V10i)))\ (ap\ (ap\ (c_2Emeasure_2Eindicator_fn\ A.27a)\ (ap \\
& V7c\ V10i))\ V9t)))) \wedge ((\forall V11t \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A.27a)\ V11t)\ (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V0m))) \Rightarrow ((ap\ V2g \\
& V11t) = (ap\ (ap\ (c_2Eextreal_2EEXTREAL_SUM_IMAGE\ ty_2Eenum_2Eenum) \\
& (\lambda V12i \in ty_2Eenum_2Eenum.(ap\ (ap\ c_2Eextreal_2Eextreal_mul \\
& (ap\ c_2Eextreal_2ENormal\ (ap\ V6z.27\ V12i)))\ (ap\ (ap\ (c_2Emeasure_2Eindicator_fn \\
& A.27a)\ (ap\ V7c\ V12i))\ V11t)))) \wedge (V3a = (ap\ (ap\ (ap\ (ap\ (c_2Elebesgue_2Epos_simple_fn_integral \\
& A.27a)\ V0m)\ V8k)\ V7c)\ V5z)) \wedge ((V4b = (ap\ (ap\ (ap\ (ap\ (c_2Elebesgue_2Epos_simple_fn_integral \\
& A.27a)\ V0m)\ V8k)\ V7c)\ V6z.27)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE \\
& ty_2Eenum_2Eenum)\ V8k)) \wedge ((\forall V13i \in ty_2Eenum_2Eenum.((p\ (ap \\
& (ap\ (c_2Ebool_2EIN\ ty_2Eenum_2Eenum)\ V13i)\ V8k)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Eenum_2E0))\ (ap\ V5z\ V13i)))))) \wedge \\
& ((\forall V14i \in ty_2Eenum_2Eenum.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eenum_2Eenum) \\
& V14i)\ V8k)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Eenum_2E0))\ (ap\ V6z.27\ V14i)))))) \wedge ((\forall V15i \in ty_2Eenum_2Eenum. \\
& (\forall V16j \in ty_2Eenum_2Eenum.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eenum_2Eenum) \\
& V15i)\ V8k)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eenum_2Eenum)\ V16j)\ V8k)) \wedge \\
& (\neg(V15i = V16j)))) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2EDISJOINT\ A.27a) \\
& (ap\ V7c\ V15i))\ (ap\ V7c\ V16j)))))) \wedge ((\forall V17i \in ty_2Eenum_2Eenum. \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Eenum_2Eenum)\ V17i)\ V8k)) \Rightarrow (p\ (ap\ (\\
& ap\ (c_2Ebool_2EIN\ (2^{A.27a}))\ (ap\ V7c\ V17i))\ (ap\ (c_2Emeasure_2Emeasurable_sets \\
& A.27a)\ V0m)))))) \wedge ((ap\ (c_2Epred_set_2EBIGUNION\ A.27a)\ (ap\ (ap \\
& (c_2Epred_set_2EIMAGE\ ty_2Eenum_2Eenum\ (2^{A.27a}))\ V7c)\ V8k)) = \\
& (ap\ (c_2Emeasure_2Em_space\ A.27a)\ V0m)))))))))
\end{aligned}$$