

thm_2Elebesgue_2Eseq_sup_def_compute
 (TMWn-
 LQcxNUWmFA4ZUtfW3TuCdR41WEiAcqU)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{4}$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{6}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num ($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (8)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (12)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap\ P\ x)) \mathbf{then} (the (\lambda x. x \in A \wedge p\ x))$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP_CLASS$

Let $c_2Erealax_2Etreall_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (15)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (16)$$

Definition 12 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})^{ty_2Erealax_2Ereal}) \quad (17)$$

Let $ty_2Eextreal_2Eextreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eextreal_2Eextreal \quad (18)$$

Let $c_2Eextreal_2ENormal : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENormal \in (ty_2Eextreal_2Eextreal^{ty_2Erealax_2Ereal}) \quad (19)$$

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Eenum_2E0 .

Definition 15 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Eenum_2Eenum.(ap\ (ap\ c_2Earithmetic_2E1))$

Definition 16 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Eenum_2Eenum.V0x$.

Definition 17 We define $c_2Eextreal_2Eextreal_of_num$ to be $\lambda V0n \in ty_2Eenum_2Eenum.(ap\ c_2Eextreal_2Eextreal_of_num)$

Let $c_2Eextreal_2Eextreal_add : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_add \in ((ty_2Eextreal_2Eextreal^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (20)$$

Let $c_2Erealax_2Etrealmult : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmult \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (21)$$

Definition 18 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 19 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_2F_21\ 2)\ (\lambda V2t \in 2))$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a})).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ ty_2Erealax_2Ereal)))$

Definition 21 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal})).(ap\ (c_2Emin_2E_40\ ty_2Erealax_2Ereal))$

Let $c_2Eextreal_2ENegInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2ENegInf \in ty_2Eextreal_2Eextreal \quad (22)$$

Definition 22 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Eextreal_2EPosInf : \iota$ be given. Assume the following.

$$c_2Eextreal_2EPosInf \in ty_2Eextreal_2Eextreal \quad (23)$$

Let $c_2Eextreal_2Eextreal_le : \iota$ be given. Assume the following.

$$c_2Eextreal_2Eextreal_le \in ((2^{ty_2Eextreal_2Eextreal})^{ty_2Eextreal_2Eextreal}) \quad (24)$$

Definition 24 We define $c_2Eextreal_2Eextreal_sup$ to be $\lambda V0p \in (2^{ty_2Eextreal_2Eextreal}). (ap (ap (ap (c_2E$

Definition 25 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 26 We define $c_2Eextreal_2Eextreal_lt$ to be $\lambda V0x \in ty_2Eextreal_2Eextreal. \lambda V1y \in ty_2Eext$

Definition 27 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let $c_2Elebesgue_2Eseq_sup : \iota$ be given. Assume the following.

$$c_2Elebesgue_2Eseq_sup \in ((ty_2Eextreal_2Eextreal)^{ty_2Enum_2Enum})^{(2^{ty_2Eextreal_2Eextreal})} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0f \in ((A_27a)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\ & (\forall V1g \in (A_27a)^{ty_2Enum_2Enum}). ((\forall V2n \in ty_2Enum_2Enum. \\ & ((ap V1g (ap c_2Enum_2ESUC V2n)) = (ap (ap V0f V2n) (ap c_2Enum_2ESUC \\ & V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 V3n))) = (ap (ap V0f (ap (ap c_2Earithmetic_2E_2D \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n)))))) \wedge \\ & (\forall V4n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 V4n))) = (ap (ap V0f (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 V4n))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 V4n))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0P \in (2^{ty_2Eextreal_2Eextreal}).((ap (ap c_2Elebesgue_2Eseq_sup \\
& V0P) c_2Enum_2E0) = (ap (c_2Emin_2E_40 ty_2Eextreal_2Eextreal) \\
& (\lambda V1r \in ty_2Eextreal_2Eextreal.(ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V1r) V0P)) (ap \\
& (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2Eextreal_sup \\
& V0P)) (ap (ap c_2Eextreal_2Eextreal_add V1r) (ap c_2Eextreal_2Eextreal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \wedge \\
& (\forall V2P \in (2^{ty_2Eextreal_2Eextreal}).(\forall V3n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Elebesgue_2Eseq_sup V2P) (ap c_2Enum_2ESUC V3n)) = \\
& (ap (c_2Emin_2E_40 ty_2Eextreal_2Eextreal) (\lambda V4r \in ty_2Eextreal_2Eextreal. \\
& (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) \\
& V4r) V2P)) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eextreal_2Eextreal_lt \\
& (ap c_2Eextreal_2Eextreal_sup V2P)) (ap (ap c_2Eextreal_2Eextreal_add \\
& V4r) (ap c_2Eextreal_2ENormal (ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2E_2F \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) \\
& (ap c_2Enum_2ESUC V3n)))))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eextreal_2Eextreal_lt \\
& (ap (ap c_2Elebesgue_2Eseq_sup V2P) V3n)) V4r)) (ap (ap c_2Eextreal_2Eextreal_lt \\
& V4r) (ap c_2Eextreal_2Eextreal_sup V2P))))))))))
\end{aligned} \tag{28}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0P \in (2^{ty_2Eextreal_2Eextreal}).((ap (ap c_2Elebesgue_2Eseq_sup \\
& V0P) c_2Enum_2E0) = (ap (c_2Emin_2E_40 ty_2Eextreal_2Eextreal) \\
& (\lambda V1r \in ty_2Eextreal_2Eextreal.(ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V1r) V0P)) (ap \\
& (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2Eextreal_sup \\
& V0P)) (ap (ap c_2Eextreal_2Eextreal_add V1r) (ap c_2Eextreal_2Eextreal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))))))\wedge \\
& ((\forall V2P \in (2^{ty_2Eextreal_2Eextreal}).(\forall V3n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Elebesgue_2Eseq_sup V2P) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V3n))) = (ap (c_2Emin_2E_40 ty_2Eextreal_2Eextreal) \\
& (\lambda V4r \in ty_2Eextreal_2Eextreal.(ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V4r) V2P)) (ap \\
& (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2Eextreal_sup \\
& V2P)) (ap (ap c_2Eextreal_2Eextreal_add V4r) (ap c_2Eextreal_2ENormal \\
& (ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V3n)))))) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Eextreal_2Eextreal_lt (ap (ap c_2Elebesgue_2Eseq_sup \\
& V2P) (ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V3n))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V4r)) \\
& (ap (ap c_2Eextreal_2Eextreal_lt V4r) (ap c_2Eextreal_2Eextreal_sup \\
& V2P))))))\wedge(\forall V5P \in (2^{ty_2Eextreal_2Eextreal}).(\forall V6n \in \\
& ty_2Enum_2Enum.((ap (ap c_2Elebesgue_2Eseq_sup V5P) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V6n))) = (ap (c_2Emin_2E_40 ty_2Eextreal_2Eextreal) \\
& (\lambda V7r \in ty_2Eextreal_2Eextreal.(ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Eextreal_2Eextreal) V7r) V5P)) (ap \\
& (ap c_2Ebool_2E_2F_5C (ap (ap c_2Eextreal_2Eextreal_lt (ap c_2Eextreal_2Eextreal_sup \\
& V5P)) (ap (ap c_2Eextreal_2Eextreal_add V7r) (ap c_2Eextreal_2ENormal \\
& (ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V6n)))))) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Eextreal_2Eextreal_lt (ap (ap c_2Elebesgue_2Eseq_sup \\
& V5P) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& V6n)))) V7r)) (ap (ap c_2Eextreal_2Eextreal_lt V7r) (ap c_2Eextreal_2Eextreal_sup \\
& V5P))))))\wedge))
\end{aligned}$$