

thm_2Elift_ieee_2Efloat_div_finite
(TMS2DXAybR8oVSYMntG6Qt94XN34aQ3h2PX)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{3}$$

Let $c_2Elift_ieee_2Eerror : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Elift_ieee_2Eerror\ A_27t\ A_27w \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \tag{4}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{5}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{6}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (ty_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) (7)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} (9)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $ty_2Ebinary_ieec_2Erounding : \iota$ be given. Assume the following.

$$nonempty ty_2Ebinary_ieec_2Erounding (10)$$

Let $c_2Ebinary_ieec_2EroundTiesToEven : \iota$ be given. Assume the following.

$$c_2Ebinary_ieec_2EroundTiesToEven \in ty_2Ebinary_ieec_2Erounding (11)$$

Let $ty_2Ebinary_ieec_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieec_2Efp_op A0 A1) (12)$$

Let $ty_2Ebinary_ieec_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieec_2Efloat A0 A1) (13)$$

Let $c_2Ebinary_ieec_2EFP_Div : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieec_2EFP_Div A_27t A_27w \in (((ty_2Ebinary_ieec_2Efp_op A_27t A_27w)(ty_2Ebinary_ieec_2Efloat A_27t A_27w))(ty_2Ebinary_ieec_2Erounding A_27t A_27w)) (14)$$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efc_2Ecart A0 A1) (15)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (16)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (23)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (24)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (25)$$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (26)$$

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_21\ 2))$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40 (A_27a) V0P))))$

Definition 18 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_Eprim_rec_2E_3C) V0m V1n)$

Definition 19 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C (A_27a) V0P)))$

Definition 20 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a) (ty_2Enum_2Enum)))$

Let $c_Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (27)$$

Definition 21 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 22 We define $c_Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap (c_Eword_msb) V0w)$

Definition 23 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EBIT2) V0n)$

Let $c_Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (28)$$

Let $c_Ereal_2Epow : \iota$ be given. Assume the following.

$$c_Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (29)$$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (30)$$

Definition 24 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_Ebool_2ECOND) V1t1 V2t2))))$

Definition 25 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2ECOND) V0b) V1n))$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum)^{ty_2Enum_2Enum}})^{ty_2Enum_2Enum}) \quad (31)$$

Definition 26 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c_2Erealax_2Etreax_2Einv\ : \iota)$ be given. Assume the following.

$$c_2Erealax_2Etreax_2Einv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (32)$$

Definition 27 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_2EABS$ Let $c_2Erealax_2Etreax_2Emul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_2Emul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (33)$$

Definition 28 We define $c_2Erealax_2Ereal_2Emul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 29 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_2EMAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_2EMAX\ A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (34)$$

Let $c_2Ebinary_2Eieee_2Efloat_2EExponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2Eieee_2Efloat_2EExponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_2Eieee_2Efloat\ A_27t\ A_27w)}) \quad (35)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (36)$$

Let $c_2Ebinary_2Eieee_2Efloat_2ESign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2Eieee_2Efloat_2ESign\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_2Eieee_2Efloat\ A_27t\ A_27w)}) \quad (37)$$

Let $c_2Erealax_2Etreax_2Eneg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_2Eneg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (38)$$

Definition 30 We define $c_2Erealax_2Ereal_2Eneg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (39)$$

Definition 31 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (40)$$

Definition 32 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 34 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 35 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 36 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcf_2EFCF$

Definition 37 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let ty_2Ebina

$$nonempty\ ty_2Ebina$$

Let c_2Ebina

$$c_2Ebina \in (ty_2Ebina^{ty_2Erealax_2Ereal}) \quad (42)$$

Let c_2Ebina

$$c_2Ebina \in ty_2Ebina \quad (43)$$

Let c_2Ebina

$$c_2Ebina \in ty_2Ebina \quad (44)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (45)$$

Definition 38 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w\ A_27a) (ap (c_2Ew$

Definition 39 We define c_2Ebina

Let c_2Ebina

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebina_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebina_ieee_2Efloat_value} \quad (46)$$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_nan)$

Definition 41 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_signalling)$

Definition 42 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27a. \dots))$

Definition 43 We define $c_2Ebinary_ieee_2Efloat_some_qnan$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0fp_op \in (ty_2Ebinary_ieee_2Efloat_some_qnan)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (47)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (48)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (49)$$

Let $ty_2Ebinary_ieee_2Eflags : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Eflags \quad (50)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (51)$$

Definition 44 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Let $c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (52)$$

Let $c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (53)$$

Let $c_2Ebinary_ieee_2Eflags_Precision_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Precision_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (54)$$

Let $c_2Ebinary_ieee_2Eflags_Overflow_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Overflow_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (55)$$

Let $c_2Ebinary_ieee_2Eflags_InvalidOp_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_InvalidOp_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (56)$$

Let $c_2Ebinary_ieee_2Eflags_DivideByZero_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_DivideByZero_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (57)$$

Definition 45 We define $c_2Ebinary_ieee_2Eclear_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_DivideByZero_fupd))$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEXISTS A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (58)$$

Definition 46 We define $c_2Ebinary_ieee_2Echeck_for_signalling$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0l \in (ty_2Ebool)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (59)$$

Definition 47 We define c_2Epair_2E2C to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod))$

Definition 48 We define $c_2Ebinary_ieee_2Einvalidop_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_InvalidOp_fupd))$

Let $c_2Ebinary_ieee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_infinity A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (60)$$

Let $c_2Ebinary_ieee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_infinity A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (61)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (62)$$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (63)$$

Let $c_2Erealx_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (64)$$

Definition 49 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Definition 50 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 51 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONI$

Let $c_2Ebinary_ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Elargest \\ & A_27t\ A_27w \in (ty_2Erealx_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (65)$$

Definition 52 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_finite)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (66)$$

Definition 53 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$

Definition 54 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 55 We define $c_2Ebinary_ieee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite)})$

Definition 56 We define $c_2Ebinary_ieee_2Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite)})$

Definition 57 We define $c_2Ebinary_ieee_2Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Ebinary_ieee_2Eclosest_such\ A_27a\ A_27b))$

Let $c_2Ebinary_ieee_2Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_top \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (67)$$

Definition 58 We define $c_Ereal_Ereal_gt$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Ebinary_ieee_Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_bottom\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (68)$$

Definition 59 We define $c_Ereal_Ereal_ge$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Ebinary_ieee_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Ethreshold\ A_27t\ A_27w \in (ty_Erealax_Ereal^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (69)$$

Definition 60 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcpcart\ 2\ A_27a).(ap$

Let $c_Ebinary_ieee_ERounding2num : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_ERounding2num \in (ty_Eenum_Eenum^{ty_Ebinary_ieee_ERounding}) \quad (70)$$

Definition 61 We define $c_Ebinary_ieee_ERounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_Ebinary_ieee_E$

Definition 62 We define $c_Ebinary_ieee_ERound$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebinary_ie$

Definition 63 We define $c_Ebinary_ieee_Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinary$

Definition 64 We define $c_Ebinary_ieee_Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebin$

Let $c_Ewords_EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_EINT_MIN\ A_27a \in (ty_Eenum_Eenum^{(ty_Ebool_Eitself\ A_27a)}) \quad (71)$$

Let $ty_ESum_ESum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_ESum_ESum\ A0\ A1) \quad (72)$$

Let $c_Ewords_Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_Edimword\ A_27a \in (ty_Eenum_Eenum^{(ty_Ebool_Eitself\ A_27a)}) \quad (73)$$

Definition 65 We define $c_Ewords_Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcpcart\ 2\ A_27a).$

Definition 66 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in$

Definition 67 We define c_Ewords_Eenzcv to be $\lambda A_27a : \iota.\lambda V0a \in (ty_Efcpcart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (74)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (75)$$

Definition 68 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 69 We define $c_2Ewords_2Eword_ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 70 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Definition 71 We define $c_2Ebinary_ieee_2Efloat_round_with_flags$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode$

Definition 72 We define $c_2Ebinary_ieee_2Edividezero_flags$ to be $(ap\ (ap\ c_2Ebina$

Definition 73 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair$

Definition 74 We define c_2Ebina

Assume the following.

$$True \quad (76)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned}
& \forall A.27t.nonempty A.27t \Rightarrow \forall A.27w.nonempty A.27w \Rightarrow (\\
& \quad \forall V0a \in (ty_2Ebinary_ieee_2Efloat A.27t A.27w). (\forall V1b \in \\
& (ty_2Ebinary_ieee_2Efloat A.27t A.27w). (((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& \quad A.27t A.27w) V0a)) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& \quad A.27t A.27w) V1b)) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_zero \\
& \quad A.27t A.27w) V1b))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs \\
& \quad (ap (ap c_2Ereal_2E_2F (ap (c_2Ebinary_ieee_2Efloat_to_real \\
& \quad A.27t A.27w) V0a)) (ap (c_2Ebinary_ieee_2Efloat_to_real A.27t \\
& \quad A.27w) V1b)))))) (ap (c_2Ebinary_ieee_2Ethreshold A.27t A.27w) \\
& \quad (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A.27t A.27w)))))) \Rightarrow \\
& \quad ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite A.27t A.27w) (\\
& \quad ap (c_2Epair_2ESND ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat \\
& \quad A.27t A.27w)) (ap (ap (ap (c_2Ebinary_ieee_2Efloat_div A.27t \\
& \quad A.27w) c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b)))) \wedge ((\\
& \quad ap (c_2Ebinary_ieee_2Efloat_to_real A.27t A.27w) (ap (c_2Epair_2ESND \\
& \quad ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat A.27t \\
& \quad A.27w)) (ap (ap (ap (c_2Ebinary_ieee_2Efloat_div A.27t A.27w) \\
& \quad c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b))) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad (ap (ap c_2Ereal_2E_2F (ap (c_2Ebinary_ieee_2Efloat_to_real \\
& \quad A.27t A.27w) V0a)) (ap (c_2Ebinary_ieee_2Efloat_to_real A.27t \\
& \quad A.27w) V1b))) (ap (ap (c_2Elift_ieee_2Eerror A.27t A.27w) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod A.27t A.27w))) (ap (ap c_2Ereal_2E_2F (ap (c_2Ebinary_ieee_2Efloat_to_real \\
& \quad A.27t A.27w) V0a)) (ap (c_2Ebinary_ieee_2Efloat_to_real A.27t \\
& \quad A.27w) V1b))))))))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (79)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (80)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (82)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (83)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{85}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{87}$$

Theorem 1

$$\begin{aligned}
& \forall A_27t. nonempty A_27t \Rightarrow \forall A_27w. nonempty A_27w \Rightarrow (\\
& \forall V0a \in (ty_2Ebinary_ieee_2Efloat A_27t A_27w). (\forall V1b \in \\
& (ty_2Ebinary_ieee_2Efloat A_27t A_27w). (((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& A_27t A_27w) V0a)) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& A_27t A_27w) V1b)) \wedge ((\neg(p (ap (c_2Ebinary_ieee_2Efloat_is_zero \\
& A_27t A_27w) V1b))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs \\
& (ap (ap c_2Ereal_2E_2F (ap (c_2Ebinary_ieee_2Efloat_to_real \\
& A_27t A_27w) V0a)) (ap (c_2Ebinary_ieee_2Efloat_to_real A_27t \\
& A_27w) V1b)))) (ap (c_2Ebinary_ieee_2Ethreshold A_27t A_27w) \\
& (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w))))))))) \Rightarrow \\
& (p (ap (c_2Ebinary_ieee_2Efloat_is_finite A_27t A_27w) (ap \\
& (c_2Epair_2ESND ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat \\
& A_27t A_27w) (ap (ap (ap (c_2Ebinary_ieee_2Efloat_div A_27t \\
& A_27w) c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b))))))
\end{aligned}$$