

thm\_2Elift\_\_ieec\_2Efloat\_\_ge  
(TMEjcMhf9Uu8pECm643Z67CQZag5xYw1T3b)

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Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (1)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EfcP\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (4)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 4** We define  $c\_Ebool\_2ET$  to be  $(ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a})))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a})))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (10)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal} \quad (12)$$

Let  $ty\_2Efcpx\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcpx\_2Ecart\ A0\ A1) \quad (13)$$

Let  $ty\_2Ebinary\_ieeex\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_ieeex\_2Efloat\ A0\ A1) \quad (14)$$

Let  $c\_2Ebinary\_ieeex\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t. nonempty\ A\_27t \Rightarrow \forall A\_27w. nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieeex\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcpx\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieeex\_2Efloat\ A\_27t\ A\_27w)}) \quad (15)$$

Let  $ty\_2Efcpx\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcpx\_2Efinite\_image\ A0) \quad (16)$$

**Definition 9** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E$

**Definition 12** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 13** We define  $c\_Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 14** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_2E\_40$

**Definition 15** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_Ebool\_2E\_2F\_5C$

**Definition 17** We define  $c\_Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}$

Let  $c\_Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Efcp\_2Edest\_cart \\ & A\_27a A\_27b \in ((A\_27a^{(ty\_2Efc\_2Efinite\_image A\_27b)})(ty\_2Efc\_2Ecart A\_27a A\_27b)) \end{aligned} \quad (17)$$

**Definition 18** We define  $c\_Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efc\_2Ecart A\_27a$

Let  $c\_Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 19** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(\mathbf{if}$

**Definition 20** We define  $c\_Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool$

Let  $c\_Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (19)$$

**Definition 21** We define  $c\_Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart 2 A\_27a).(ap (ap c$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (20)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod \\ & A0 A1) \end{aligned} \quad (21)$$

Let  $c\_Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (22)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (t$   
Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (23)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (24)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}})) \quad (25)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 24** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (26)$$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 26** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

**Definition 27** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (27)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (28)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent A\_27t A\_27w \in ((ty\_2EfcP\_2Ecart 2 A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (29)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (30)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (31)$$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (32)$$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 30** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (35)$$

**Definition 31** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 33** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 34** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 35** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Efc\_2EFCP$

**Definition 36** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (37)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (38)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (39)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in ( ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)} ) \quad (40)$$

**Definition 37** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (c\_2Ew$

**Definition 38** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})_{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (41)$$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_compare : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (42)$$

Let  $c\_2Ebinary\_ieee\_2EUN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EUN \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (43)$$

Let  $c\_2Ebinary\_ieee\_2EGT : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EGT \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (44)$$

Let  $c\_2Ebinary\_ieee\_2EEQ : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EEQ \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (45)$$

Let  $c\_2Ebinary\_ieee\_2ELT : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ELT \in ty\_2Ebinary\_ieee\_2Efloat\_compare \quad (46)$$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (47)$$

**Definition 40** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$ .

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_Epair\_Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (48)$$

**Definition 41** We define  $c\_Epair\_E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_E\_2C\ x\ y))$ .

Let  $c\_Epair\_EESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (49)$$

Let  $c\_Epair\_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EFAST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (50)$$

**Definition 42** We define  $c\_Epair\_Epair\_CASE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_Epair\_Epair\_CASE\ A\_27a\ A\_27b\ A\_27c\ p)$ .

**Definition 43** We define  $c\_Ebinary\_ieee\_Efloat\_compare$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_Ebinary\_ieee\_Efloat\_compare\ A\_27t\ A\_27w\ x)$ .

Let  $c\_Ebinary\_ieee\_Efloat\_compare2num : \iota$  be given. Assume the following.

$$c\_Ebinary\_ieee\_Efloat\_compare2num \in (ty\_Enum\_Enum^{ty\_Ebinary\_ieee\_Efloat\_compare}) \quad (51)$$

**Definition 44** We define  $c\_Ebinary\_ieee\_Efloat\_compare\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_Ebinary\_ieee\_Efloat\_compare\ A\_27a\ x$ .

**Definition 45** We define  $c\_Ebinary\_ieee\_Efloat\_greater\_equal$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_Ebinary\_ieee\_Efloat\_greater\_equal\ A\_27t\ A\_27w\ x)$ .

Let  $c\_Earithmetic\_EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEVEN \in (2^{ty\_Enum\_Enum}) \quad (52)$$

Let  $c\_Earithmetic\_EODD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EODD \in (2^{ty\_Enum\_Enum}) \quad (53)$$

**Definition 46** We define  $c\_Earithmetic\_E\_3E$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$ .

**Definition 47** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ t1\ t2)))$ .

**Definition 48** We define  $c\_Earithmetic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$ .

**Definition 49** We define  $c\_Eprim\_rec\_EPRE$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ (ap\ (ap\ (c\_Ebool\_E\_21\ 2)\ m)\ n))$ .

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (54)$$

**Definition 50** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 51** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$ .

**Definition 52** We define  $c\_2Ereal\_2Ereal\_ge$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$ .

**Definition 53** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$ .

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (55)$$

**Definition 54** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcpc\_2Ecart\ 2\ A\_27a)$ .

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ c\_2Enum\_2E0)\ V0n))) \quad (56)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m)\ V1n)))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E3C\ V1n)\ V0m)))) \quad (57)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0)))) \quad (58)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0n)\ c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ (ap\ c\_2Enum\_2ESUC\ V2n)))) \Leftrightarrow ((V1m = (ap\ c\_2Enum\_2ESUC\ V2n)) \vee (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ V2n))))))) \quad (59)$$



Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0a \in \text{ty\_2Erealax\_2Ereal}. \\
& \quad (\forall V1f \in (A\_27a^{\text{ty\_2Erealax\_2Ereal}}).\forall V2v \in A\_27a. \\
& \quad (\forall V3v1 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_value\_CASE} \\
& \quad \quad A\_27a) (\text{ap } \text{c\_2Ebinary\_ieee\_2Efloat } V0a)) V1f) V2v) V3v1) = (\text{ap} \\
& \quad \quad V1f V0a)))))) \wedge ((\forall V4f \in (A\_27a^{\text{ty\_2Erealax\_2Ereal}}).\forall V5v \in \\
& \quad A\_27a.(\forall V6v1 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_value\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2EInfinity}) V4f) V5v) V6v1) = V5v)))) \wedge \\
& \quad (\forall V7f \in (A\_27a^{\text{ty\_2Erealax\_2Ereal}}).\forall V8v \in A\_27a. \\
& \quad (\forall V9v1 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_value\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2ENaN}) V7f) V8v) V9v1) = V9v1)))))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0v0 \in A\_27a.(\forall V1v1 \in \\
& \quad A\_27a.(\forall V2v2 \in A\_27a.(\forall V3v3 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_compare\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2ELT}) \\
& \quad \quad V0v0) V1v1) V2v2) V3v3) = V0v0)))))) \wedge ((\forall V4v0 \in A\_27a.(\forall V5v1 \in \\
& \quad A\_27a.(\forall V6v2 \in A\_27a.(\forall V7v3 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_compare\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2EEQ}) \\
& \quad \quad V4v0) V5v1) V6v2) V7v3) = V5v1)))))) \wedge ((\forall V8v0 \in A\_27a.(\forall V9v1 \in \\
& \quad A\_27a.(\forall V10v2 \in A\_27a.(\forall V11v3 \in A\_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_compare\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2EGT}) \\
& \quad \quad V8v0) V9v1) V10v2) V11v3) = V10v2)))))) \wedge ((\forall V12v0 \in A\_27a.(\forall V13v1 \in A\_27a.(\forall V14v2 \in A\_27a.(\forall V15v3 \in A\_27a. \\
& \quad ((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Ebinary\_ieee\_2Efloat\_compare\_CASE} \\
& \quad \quad A\_27a) \text{c\_2Ebinary\_ieee\_2EUN}) V12v0) V13v1) V14v2) V15v3) = V15v3))))))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (62)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \hspace{15em} (64)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \hspace{15em} (65)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge \\
& \quad ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.(((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow \quad (76) \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27}))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\
& (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c.2Ebool.2ECOND A_{.27a}) \\
& V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool.2ECOND A_{.27a}) V1Q) V3x_{.27}) \\
& V5y_{.27}))))))))) \quad (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & \quad nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & \quad ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair\_CASE\ A.27a\ A.27b \\ & \quad A.27c)\ (ap\ (ap\ (c.2Epair.2E\_2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & \quad (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & ((ap\ (ap\ c.2Erealx\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & (\forall V2z \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad V0x)\ (ap\ (ap\ c.2Erealx\_2Ereal\_add\ V1y)\ V2z)) = (ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad (ap\ (ap\ c.2Erealx\_2Ereal\_add\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad (ap\ c.2Ereal.2Ereal\_of\_num\ c.2Enum.2E0))\ V0x) = V0x)) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad (ap\ c.2Erealx\_2Ereal\_neg\ V0x))\ V0x) = (ap\ c.2Ereal.2Ereal\_of\_num \\ & \quad c.2Enum.2E0))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & (\forall V2z \in ty\_2Erealx\_2Ereal. (((p\ (ap\ (ap\ c.2Erealx\_2Ereal\_lt \\ & \quad V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Erealx\_2Ereal\_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap \\ & \quad (ap\ c.2Erealx\_2Ereal\_lt\ V0x)\ V2z)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c.2Erealx\_2Ereal\_mul \\ & \quad (ap\ c.2Ereal.2Ereal\_of\_num\ (ap\ c.2Earithmetic.2ENUMERAL\ ( \\ & \quad ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ V0x) = V0x)) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. ((ap\ (ap\ c.2Erealx\_2Ereal\_add \\ & \quad V0x)\ (ap\ c.2Ereal.2Ereal\_of\_num\ c.2Enum.2E0)) = V0x)) \end{aligned} \quad (86)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V2z)))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z)))))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V2z)))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V2z)))))) \quad (92)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z)))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V2z)))))) \quad (93)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (94)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num \\
V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0y) V1x))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
V1y) V2z)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V2z))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
(ap c\_2Erealax\_2Ereal\_neg V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0m) V1n))))
\end{aligned} \tag{104}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{105}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{108}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False))) \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{110}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \tag{111}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \tag{112}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
& \tag{113}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \\
& \tag{114}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((ap (c.2Ewords.2Eword\_2comp \\
& A.27a) (ap (c.2Ewords.2En2w A.27a) (ap c.2Earithmetic.2ENUMERAL \\
& (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))) = (c.2Ewords.2Eword\_T \\
& A.27a)) \\
& \tag{115}
\end{aligned}$$

### Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\
& \forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat A.27a A.27b). (\forall V1y \in \\
& (ty\_2Ebinary\_ieee\_2Efloat A.27a A.27b). (((p (ap (c.2Ebinary\_ieee\_2Efloat\_is\_finite \\
& A.27a A.27b) V0x)) \wedge (p (ap (c.2Ebinary\_ieee\_2Efloat\_is\_finite \\
& A.27a A.27b) V1y))) \Rightarrow ((p (ap (ap (c.2Ebinary\_ieee\_2Efloat\_greater\_equal \\
& A.27a A.27b) V0x) V1y)) \Leftrightarrow (p (ap (ap c.2Ereal.2Ereal\_ge (ap (c.2Ebinary\_ieee\_2Efloat\_to\_real \\
& A.27a A.27b) V0x)) (ap (c.2Ebinary\_ieee\_2Efloat\_to\_real A.27a \\
& A.27b) V1y))))))))) \\
& \tag{116}
\end{aligned}$$