

thm_2Elift_ieee_2Efloat_le
(TMQ8pP9XiEssQJXiPvgAhC5FHPA xhjyg8BB)

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Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (3)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EfcP_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (4)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 4 We define c_Ebool_2ET to be $(ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_Emin_2E_3D (2^{A_27a}))))$

Definition 6 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num (ap (ap (c_Emin_2E_3D (2^{A_27a}))))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B (ap (ap (c_Emin_2E_3D (2^{A_27a}))))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (10)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (12)$$

Let $ty_2Efcpx_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcpx_2Ecart\ A0\ A1) \quad (13)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (14)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t. nonempty\ A_27t \Rightarrow \forall A_27w. nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efcpx_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (15)$$

Let $ty_2Efcpx_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcpx_2Efinite_image\ A0) \quad (16)$$

Definition 9 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (23)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (24)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}})) \quad (25)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 25 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 26 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 28 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MAX A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (28)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent A_27t A_27w \in ((ty_2EfcP_2Ecart 2 A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (29)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (30)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign \\ & A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (31)$$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$\begin{aligned} & c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (32)$$

Definition 29 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 30 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (34)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 31 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 33 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 34 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 35 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP$

Definition 36 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (37)$$

Let $c_2Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (38)$$

Let $c_2Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (39)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (40)$$

Definition 37 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Ew$

Definition 38 We define $c_2Ebinary_ieee_2Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})_{ty_2Ebinary_ieee_2Efloat_value} \quad (41)$$

Definition 39 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $ty_2Ebinary_ieee_2Efloat_compare : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_compare \quad (42)$$

Let $c_2Ebinary_ieee_2EUN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EUN \in ty_2Ebinary_ieee_2Efloat_compare \quad (43)$$

Let $c_2Ebinary_ieee_2EGT : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EGT \in ty_2Ebinary_ieee_2Efloat_compare \quad (44)$$

Let $c_2Ebinary_ieee_2EEQ : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EEQ \in ty_2Ebinary_ieee_2Efloat_compare \quad (45)$$

Let $c_2Ebinary_ieee_2ELT : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ELT \in ty_2Ebinary_ieee_2Efloat_compare \quad (46)$$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (47)$$

Definition 40 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$.

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (48)$$

Definition 41 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_E_2C\ x\ y))$.

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EESND \\ A_27a\ A_27b \in (A_27b^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (49)$$

Let $c_Epair_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EFAST \\ A_27a\ A_27b \in (A_27a^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (50)$$

Definition 42 We define $c_Epair_Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_Epair_Epair_CASE\ A_27a\ A_27b\ A_27c)$.

Definition 43 We define $c_Ebinary_ieee_Efloat_compare$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinary_ieee_Efloat_compare\ A_27t\ A_27w)$.

Let $c_Ebinary_ieee_Efloat_compare2num : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_Efloat_compare2num \in (ty_Enum_Enum^{ty_Ebinary_ieee_Efloat_compare}) \quad (51)$$

Definition 44 We define $c_Ebinary_ieee_Efloat_compare_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_Ebinary_ieee_Efloat_compare\ A_27a$.

Definition 45 We define $c_Ebinary_ieee_Efloat_less_equal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinary_ieee_Efloat_less_equal\ A_27t\ A_27w)$.

Let $c_Earithmetic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_EEVEN \in (2^{ty_Enum_Enum}) \quad (52)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_Enum_Enum}) \quad (53)$$

Definition 46 We define $c_Earithmetic_E_3E$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$.

Definition 47 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ t1\ t2)))$.

Definition 48 We define $c_Earithmetic_E_3E_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$.

Definition 49 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_Enum_Enum.(ap\ (ap\ (ap\ (c_Ebool_E_21\ 2)\ t1\ t2)))$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (54)$$

Definition 50 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 51 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 52 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (55)$$

Definition 53 We define $c_2Ewords_2Eword_comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a)$.

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ c_2Enum_2E0)\ V0n))) \quad (56)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V1n)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E3C\ V1n)\ V0m)))) \quad (57)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0)))) \quad (58)$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ (ap\ c_2Enum_2ESUC\ V2n))) \Leftrightarrow ((V1m = (ap\ c_2Enum_2ESUC\ V2n)) \vee (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V2n))))))) \quad (59)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in ty_2Erealax_2Ereal. \\
& \quad (\forall V1f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V2v \in A_27a. \\
& \quad (\forall V3v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& \quad A_27a)\ (ap\ c_2Ebinary_ieee_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\
& \quad V1f\ V0a)))))) \wedge ((\forall V4f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V5v \in \\
& A_27a.(\forall V6v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& A_27a)\ c_2Ebinary_ieee_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\
& \quad (\forall V7f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V8v \in A_27a. \\
& (\forall V9v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\
& A_27a)\ c_2Ebinary_ieee_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1)))))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0v0 \in A_27a.(\forall V1v1 \in \\
& \quad A_27a.(\forall V2v2 \in A_27a.(\forall V3v3 \in A_27a.((ap\ (ap\ (ap\ (\\
& \quad ap\ (ap\ (c_2Ebinary_ieee_2Efloat_compare_CASE\ A_27a)\ c_2Ebinary_ieee_2ELT) \\
& \quad V0v0)\ V1v1)\ V2v2)\ V3v3) = V0v0)))))) \wedge ((\forall V4v0 \in A_27a.(\forall V5v1 \in \\
& \quad A_27a.(\forall V6v2 \in A_27a.(\forall V7v3 \in A_27a.((ap\ (ap\ (ap\ (\\
& \quad ap\ (ap\ (c_2Ebinary_ieee_2Efloat_compare_CASE\ A_27a)\ c_2Ebinary_ieee_2EEQ) \\
& \quad V4v0)\ V5v1)\ V6v2)\ V7v3) = V5v1)))))) \wedge ((\forall V8v0 \in A_27a.(\forall V9v1 \in \\
& \quad A_27a.(\forall V10v2 \in A_27a.(\forall V11v3 \in A_27a.((ap\ (ap\ (ap\ (\\
& \quad (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_compare_CASE\ A_27a)\ c_2Ebinary_ieee_2EGT) \\
& \quad V8v0)\ V9v1)\ V10v2)\ V11v3) = V10v2)))))) \wedge ((\forall V12v0 \in A_27a.(\\
& \quad \forall V13v1 \in A_27a.(\forall V14v2 \in A_27a.(\forall V15v3 \in A_27a. \\
& \quad ((ap\ (ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_compare_CASE \\
& \quad A_27a)\ c_2Ebinary_ieee_2EUN)\ V12v0)\ V13v1)\ V14v2)\ V15v3) = V15v3)))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (62)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \hspace{15em} (64)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \hspace{15em} (65)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge \\
& \quad ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow \quad (76) \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27}))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\
& (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c.2Ebool.2ECOND A_{.27a}) \\
& V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool.2ECOND A_{.27a}) V1Q) V3x_{.27}) \\
& V5y_{.27}))))))))) \quad (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & \quad nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & \quad ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair_CASE\ A.27a\ A.27b \\ & \quad A.27c)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & \quad (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & ((ap\ (ap\ c.2Erealx_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c.2Erealx_2Ereal_add \\ & \quad V1y)\ V0x)))) \end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & (\forall V2z \in ty_2Erealx_2Ereal. ((ap\ (ap\ c.2Erealx_2Ereal_add \\ & V0x)\ (ap\ (ap\ c.2Erealx_2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c.2Erealx_2Ereal_add \\ & \quad (ap\ (ap\ c.2Erealx_2Ereal_add\ V0x)\ V1y))\ V2z)))))) \end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c.2Erealx_2Ereal_add \\ & (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V0x) = V0x)) \end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c.2Erealx_2Ereal_add \\ & (ap\ c.2Erealx_2Ereal_neg\ V0x))\ V0x) = (ap\ c.2Ereal.2Ereal_of_num \\ & \quad c.2Enum.2E0))) \end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\ & (\forall V2z \in ty_2Erealx_2Ereal. (((p\ (ap\ (ap\ c.2Erealx_2Ereal_lt \\ & V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap \\ & \quad (ap\ c.2Erealx_2Ereal_lt\ V0x)\ V2z)))))) \end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c.2Erealx_2Ereal_mul \\ & (ap\ c.2Ereal.2Ereal_of_num\ (ap\ c.2Earithmetic.2ENUMERAL\ (\\ & \quad ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ V0x) = V0x)) \end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c.2Erealx_2Ereal_add \\ & V0x)\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0)) = V0x)) \end{aligned} \tag{86}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (92)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (93)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z)))))) \quad (94)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x)))) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V2z)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0m) V1n))))
\end{aligned} \tag{105}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{106}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B)))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B)))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{109}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False))) \tag{110}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((ap (c.2Ewords.2Eword_2comp \\
& A.27a) (ap (c.2Ewords.2En2w A.27a) (ap c.2Earithmetic.2ENUMERAL \\
& (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))) = (c.2Ewords.2Eword_T \\
& A.27a))
\end{aligned} \tag{116}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\
& \forall V0x \in (ty_2Ebinary_ieee_2Efloat A.27a A.27b). (\forall V1y \in \\
& (ty_2Ebinary_ieee_2Efloat A.27a A.27b). (((p (ap (c.2Ebinary_ieee_2Efloat_is_finite \\
& A.27a A.27b) V0x)) \wedge (p (ap (c.2Ebinary_ieee_2Efloat_is_finite \\
& A.27a A.27b) V1y))) \Rightarrow ((p (ap (ap (c.2Ebinary_ieee_2Efloat_less_equal \\
& A.27a A.27b) V0x) V1y)) \Leftrightarrow (p (ap (ap c.2Ereal.2Ereal_lte (ap (c.2Ebinary_ieee_2Efloat_to_real \\
& A.27a A.27b) V0x)) (ap (c.2Ebinary_ieee_2Efloat_to_real A.27a \\
& A.27b) V1y))))))
\end{aligned}$$