thm_2Elift__ieee_2Efloat__sqrt__finite (TMZaAtWGXSPMqBwTzaidi8VQfhX7LCtPHUv)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (1)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (3)$$

Let $c\_2Elift\_\_ieee\_2Eerror : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Elift\_\_ieee\_2Eerror\ A\_27t\ A\_27w \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (4)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (5)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (6)$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{7}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{8}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{9}$$

Definition 6 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 7 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $ty\_2Ebinary\_\_ieee\_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Erounding \tag{10}$$

Let $c\_2Ebinary\_\_ieee\_2EroundTiesToEven : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EroundTiesToEven \in ty\_2Ebinary\_\_ieee\_2Erounding \tag{11}$$

Let $ty\_2Ebinary\_\_ieee\_2Efp\_\_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efp\_\_op\ A0\ A1) \tag{12}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{13}$$

Let $c\_2Ebinary\_\_ieee\_2EFP\_\_Sqrt : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2EFP\_\_Sqrt\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efp\_\_op\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{ty\_2Ebi} \tag{14}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{15}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{16}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{17}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{18}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{19}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{21}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{23}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{24}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{25}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{26}$$

Definition 13 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 16 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 18 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 19 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 20 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \quad (27)$$

Definition 21 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 22 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 23 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (28)$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (29)$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (30)$$

Definition 24 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 25 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Eboo}$

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (31)$$

Definition 26 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (32)$$

Definition 27 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (33)$$

Definition 28 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 29 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Ewords\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2} \quad (34)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (35)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (36)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (37)$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (38)$$

Definition 30 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (39)$$

Definition 31 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (40)$$

Definition 32 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 33 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 34 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 35 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 36 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 37 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $ty\_2Ebinary\_\_ieee\_2Efloat\_\_value : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \quad (41)$$

Let $c\_2Ebinary\_\_ieee\_2EFloat : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EFloat \in (ty\_2Ebinary\_\_ieee\_2Efloat\_\_value^{ty\_2Erealax\_2Ereal}) \quad (42)$$

Let $c\_2Ebinary\_\_ieee\_2ENaN : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2ENaN \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \quad (43)$$

Let $c\_2Ebinary\_\_ieee\_2EInfinity : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EInfinity \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \quad (44)$$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (45)$$

Definition 38 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ewords_2En2w A_27a) $(ap\ (c\_2Ew$

Definition 39 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE\ A\_27a \in ((((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_\_ieee\_2Efloat\_\_value}) \quad (46)$$

Definition 40 *We define* c_2Ebinary__ieee_2Efloat__is__nan *to be* $\lambda A\_27t:\iota.\lambda A\_27w:\iota.\lambda V0x\in(ty\_2Ebinar$

Definition 41 *We define* c_2Ebinary__ieee_2Efloat__is__signalling *to be* $\lambda A\_27t:\iota.\lambda A\_27w:\iota.\lambda V0x\in(ty\_2Eb$

Definition 42 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0f\in(A\_27b^{A\_27a}).(\lambda V1x\in A\_27$

Definition 43 *We define* c_2Ebinary__ieee_2Efloat__some__qnan *to be* $\lambda A\_27t:\iota.\lambda A\_27w:\iota.\lambda V0fp\_\_op\in(ty$

Let $ty\_2Ebinary\_\_ieee\_2Eflags:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Eflags \tag{47}$$

Let $c\_2Ebool\_2EARB:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ebool\_2EARB\ A\_27a\in A\_27a \tag{48}$$

Definition 44 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0x\in A\_27a.(\lambda V1y\in A\_27b.V0x)$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_AfterRounding\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_AfterRounding\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{49}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_BeforeRounding\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Underflow\_\_BeforeRounding\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{50}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Precision\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Precision\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{51}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_Overflow\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_Overflow\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)}) \tag{52}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_InvalidOp\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_InvalidOp\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags})^{(2^2)} \tag{53}$$

Let $c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideByZero\_\_fupd:\iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideByZero\_\_fupd\in((ty\_2Ebinary\_\_ieee\_2Eflags^{ty\_2Ebinary\_\_ieee\_2Eflags} \tag{54}$$

Definition 45 *We define* c_2Ebinary__ieee_2Eclear__flags *to be* $(ap\ (ap\ c\_2Ebinary\_\_ieee\_2Eflags\_\_DivideB$

Definition 46 *We define* c_2Ebinary__ieee_2Einvalidop__flags *to be* $(ap\ (ap\ c\_2Ebinary\_\_ieee\_2Eflags\_\_Inva$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (55)$$

Definition 47 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (56)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (57)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (58)$$

Let $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (59)$$

Definition 48 *We define* c_2Ebinary__ieee_2Echeck__for__signalling *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_2E$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (60)$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (61)$$

Definition 49 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 50 *We define* c_2Etransc_2Eroot *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1x \in ty\_2Erealax\_2Ereal$

Definition 51 *We define* c_2Etransc_2Esqrt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap$ c_2Etransc_2Eroot (

Definition 52 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 53 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Ebinary\_\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{62}$$

Definition 54 *We define* c_2Ebinary__ieee_2Efloat__is__finite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{63}$$

Definition 55 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 56 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 57 *We define* c_2Ebinary__ieee_2Eis__closest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_\_ie}$

Definition 58 *We define* c_2Ebinary__ieee_2Eclosest__such *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary}$

Definition 59 *We define* c_2Ebinary__ieee_2Eclosest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap$ (c_2Ebinary__ieee_2Eclose

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_top\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{64}$$

Definition 60 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{65}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{66}$$

Definition 61 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Ebinary\_\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (67)$$

Definition 62 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $c\_2Ebinary\_\_ieee\_2Erounding2num : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_\_ieee\_2Erounding}) \quad (68)$$

Definition 63 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_\_ieee\_2$

Definition 64 *We define* c_2Ebinary__ieee_2Eround *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_\_ie$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (69)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (70)$$

Definition 65 *We define* c_2Ebinary__ieee_2Efloat__is__zero *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 66 *We define* c_2Ebinary__ieee_2Efloat__round *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebina$

Let $c\_2Ewords\_2EINT\_\_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27} \quad (71)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (72)$$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (73)$$

Definition 67 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 68 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 69 *We define* c_2Ewords_2Enzcv *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{74}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{75}$$

Definition 70 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 71 *We define* c_2Ewords_2Eword__ls *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 72 *We define* c_2Ebinary__ieee_2Efloat__is__infinite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

Definition 73 *We define* c_2Ebinary__ieee_2Efloat__round__with__flags *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

Definition 74 *We define* c_2Ebinary__ieee_2Efloat__sqrt *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinar$

Assume the following.

$$True \tag{76}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{77}$$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (\forall V0a \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).(((p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ V0a)) \wedge (((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ V0a) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ ty\_2Eone\_2Eone)\ \mathsf{c\_2Enum\_2E0})) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ \mathsf{c\_2Etransc\_2Esqrt}\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ V0a))))\ (ap\ (c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Epair\_2ESND\ ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w))\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_sqrt}\ A\_27t\ A\_27w)\ c\_2Ebinary\_\_ieee\_2EroundTiesToEven)\ V0a)))) \wedge ((ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ (ap\ (c\_2Epair\_2ESND\ ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w))\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_sqrt}\ A\_27t\ A\_27w)\ c\_2Ebinary\_\_ieee\_2EroundTiesToEven)\ V0a))) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Etransc\_2Esqrt}\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ V0a)))\ (ap\ (ap\ (c\_2Elift\_\_ieee\_2Eerror\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))\ (ap\ \mathsf{c\_2Etransc\_2Esqrt}\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ V0a)))))))) \tag{78}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{87}$$

Theorem 1

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow(\forall V0a \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).(((p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ V0a))\wedge(((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ V0a) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ ty\_2Eone\_2Eone)\ \mathsf{c\_2Enum\_2E0}))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ \mathsf{c\_2Etransc\_2Esqrt}\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_to\_\_real}\ A\_27t\ A\_27w)\ V0a))))\ (ap\ (c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))))\Rightarrow(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Epair\_2ESND\ ty\_2Ebinary\_\_ieee\_2Eflags\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w))\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_sqrt}\ A\_27t\ A\_27w)\ c\_2Ebinary\_\_ieee\_2EroundTiesToEven)\ V0a))))))$$