

thm_2Elift_ieee_2Efloat_sub_finite
(TMQeoE3dAHaQUYpSdFLwBTZkCz8g7Gogpyq)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{3}$$

Let $c_2Elift_ieee_2Eerror : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Elift_ieee_2Eerror\ A_27t\ A_27w \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \tag{4}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{5}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{6}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (ty_2Erealax_2Ereal_add) (ty_2Erealax_2Ereal_eq)))$.
Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (7)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal))} \quad (9)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$.

Definition 7 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $ty_2Ebinary_ieee_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Erounding \quad (10)$$

Let $c_2Ebinary_ieee_2EroundTiesToEven : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EroundTiesToEven \in ty_2Ebinary_ieee_2Erounding \quad (11)$$

Let $ty_2Ebinary_ieee_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efp_op\ A0\ A1) \quad (12)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (13)$$

Let $c_2Ebinary_ieee_2EFP_Sub : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Sub\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w) (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)) (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)) \quad (14)$$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (15)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (16)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (23)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (24)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (25)$$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (26)$$

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_21\ 2))$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40$

Definition 18 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C$

Definition 20 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $c_Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Efcp_2Edest_cart \\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \end{aligned} \quad (27)$$

Definition 21 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 22 We define $c_Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap$

Definition 23 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic$

Let $c_Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (28)$$

Let $c_Ereal_2Epow : \iota$ be given. Assume the following.

$$c_Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (29)$$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (30)$$

Definition 24 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 25 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum)^{ty_2Enum_2Enum}})^{ty_2Enum_2Enum}) \quad (31)$$

Definition 26 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c$
Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etreal_inv \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (32)$$

Definition 27 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$
Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etreal_mul \in & (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (33)$$

Definition 28 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 29 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (34)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent \\ A_27t\ A_27w \in & ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (35)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (36)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign \\ A_27t\ A_27w \in & ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (37)$$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etreal_neg \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (38)$$

Definition 30 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (39)$$

Definition 31 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (40)$$

Definition 32 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 33 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 34 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 35 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 36 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcf_2EFCF$

Definition 37 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let ty_2Ebina

$$nonempty\ ty_2Ebina$$

Let c_2Ebina

$$c_2Ebina \in (ty_2Ebina^{ty_2Erealax_2Ereal}) \quad (42)$$

Let c_2Ebina

$$c_2Ebina \in ty_2Ebina \quad (43)$$

Let c_2Ebina

$$c_2Ebina \in ty_2Ebina \quad (44)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (45)$$

Definition 38 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w\ A_27a) (ap (c_2Ew$

Definition 39 We define c_2Ebina

Let c_2Ebina

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebina_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebina_ieee_2Efloat_value} \quad (46)$$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_nan)$

Definition 41 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_signalling)$

Definition 42 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27a. c_2Ebool_2ELET f x)))$

Definition 43 We define $c_2Ebinary_ieee_2Efloat_some_qnan$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0fp_op \in (ty_2Ebinary_ieee_2Efloat_some_qnan)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (47)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (48)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (49)$$

Let $ty_2Ebinary_ieee_2Eflags : \iota$ be given. Assume the following.

$$nonempty ty_2Ebinary_ieee_2Eflags \quad (50)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (51)$$

Definition 44 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Let $c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (52)$$

Let $c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (53)$$

Let $c_2Ebinary_ieee_2Eflags_Precision_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Precision_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (54)$$

Let $c_2Ebinary_ieee_2Eflags_Overflow_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Overflow_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (55)$$

Let $c_2Ebinary_ieee_2Eflags_InvalidOp_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_InvalidOp_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (56)$$

Let $c_2Ebinary_ieee_2Eflags_DivideByZero_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_DivideByZero_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (57)$$

Definition 45 We define $c_2Ebinary_ieee_2Eclear_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_DivideByZero_fupd))$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEXISTS A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (58)$$

Definition 46 We define $c_2Ebinary_ieee_2Echeck_for_signalling$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0l \in (ty_2Ebinary_ieee_2Eflags)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (59)$$

Definition 47 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod))$

Definition 48 We define $c_2Ebinary_ieee_2Einvalidop_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_InvalidOp_fupd))$

Definition 49 We define $c_2Ewords_2Eword_1comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efc_2Ecart 2 A_27a)$.

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd A_27t A_27w \in (((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)})^{(ty_2Efloat A_27t A_27w)}) \quad (60)$$

Definition 50 We define $c_2Ebinary_ieee_2Efloat_negate$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat A_27t A_27w)$

Definition 51 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2EroundTowardNegative : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EroundTowardNegative \in ty_2Ebinary_ieee_2Erounding \quad (61)$$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (62)$$

Definition 52 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 53 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 54 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONV))))$

Let $c_2Ebinary_ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Elargest \\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (63)$$

Definition 55 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Efloat_is_finite)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (64)$$

Definition 56 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 57 We define $c_2Ebinary_ieee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite)})$

Definition 58 We define $c_2Ebinary_ieee_2Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary_ieee_2Efloat_is_finite)})$

Definition 59 We define $c_2Ebinary_ieee_2Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Ebinary_ieee_2Eclosest_such))$

Let $c_2Ebinary_ieee_2Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_top \\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (65)$$

Definition 60 We define $c_2Ereal_2Ereal_gt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_bottom \\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (66)$$

Let $c_2Ebinary_ieee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_infinity \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (67)$$

Definition 61 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_infinity \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (68)$$

Let $c_2Ebinary_ieee_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Ethreshold \\ & A_27t\ A_27w \in (ty_2Erealax_2Ereal)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (69)$$

Definition 62 We define $c_2Ewords_2Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcpr_2Ecart\ 2\ A_27a).(ap$

Let $c_2Ebinary_ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erounding2num \in (ty_2Enum_2Enum)^{ty_2Ebinary_ieee_2Erounding} \quad (70)$$

Definition 63 We define $c_2Ebinary_ieee_2Erounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2E$

Definition 64 We define $c_2Ebinary_ieee_2Eround$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_iee$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (71)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (72)$$

Definition 65 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Definition 66 We define $c_2Ebinary_ieee_2Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebin$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (73)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (74)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (75)$$

Definition 67 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 68 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Definition 69 We define $c_2Ewords_2Enzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (76)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (77)$$

Definition 70 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 71 We define $c_2Ewords_2Eword_2ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Definition 72 We define $c_2Ebinary_2ieee_2Efloat_2is_2infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Definition 73 We define $c_2Ebinary_2ieee_2Efloat_2round_2with_2flags$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ($

Definition 74 We define $c_2Epair_2Epair_2CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair$

Definition 75 We define $c_2Ebinary_2ieee_2Efloat_2sub$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary$

Assume the following.

$$True \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27t}. \text{nonempty } A_{27t} \Rightarrow \forall A_{27w}. \text{nonempty } A_{27w} \Rightarrow (\\
& \quad \forall V0a \in (\text{ty_2Ebinary_ieee_2Efloat } A_{27t} A_{27w}). (\forall V1b \in \\
& (\text{ty_2Ebinary_ieee_2Efloat } A_{27t} A_{27w}). (((p \text{ (ap (c_2Ebinary_ieee_2Efloat_is_finite} \\
& \quad A_{27t} A_{27w}) V0a)) \wedge ((p \text{ (ap (c_2Ebinary_ieee_2Efloat_is_finite} \\
& \quad A_{27t} A_{27w}) V1b)) \wedge (p \text{ (ap (ap c_2Erealx_2Ereal_lt (ap c_2Ereal_2Eabs} \\
& \quad (\text{ap (ap c_2Ereal_2Ereal_sub (ap (c_2Ebinary_ieee_2Efloat_to_real} \\
& \quad A_{27t} A_{27w}) V0a)) \text{ (ap (c_2Ebinary_ieee_2Efloat_to_real } A_{27t} \\
& \quad A_{27w}) V1b)))) \text{ (ap (c_2Ebinary_ieee_2Ethreshold } A_{27t} A_{27w}) \\
& \quad (\text{c_2Ebool_2Ethe_value (ty_2Epair_2Eprod } A_{27t} A_{27w)))))) \Rightarrow \\
& \quad ((p \text{ (ap (c_2Ebinary_ieee_2Efloat_is_finite } A_{27t} A_{27w}) (} \\
& \text{ap (c_2Epair_2ESND ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat} \\
& \quad A_{27t} A_{27w}) \text{ (ap (ap (ap (c_2Ebinary_ieee_2Efloat_sub } A_{27t} \\
& \quad A_{27w}) c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b)))) \wedge ((\\
& \text{ap (c_2Ebinary_ieee_2Efloat_to_real } A_{27t} A_{27w}) \text{ (ap (c_2Epair_2ESND} \\
& \quad \text{ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat } A_{27t} \\
& \quad A_{27w}) \text{ (ap (ap (ap (c_2Ebinary_ieee_2Efloat_sub } A_{27t} A_{27w}) \\
& \quad c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b)))) = (\text{ap (ap c_2Erealx_2Ereal_add} \\
& \quad (\text{ap (ap c_2Ereal_2Ereal_sub (ap (c_2Ebinary_ieee_2Efloat_to_real} \\
& \quad A_{27t} A_{27w}) V0a)) \text{ (ap (c_2Ebinary_ieee_2Efloat_to_real } A_{27t} \\
& \quad A_{27w}) V1b)) \text{ (ap (ap (c_2Elift_ieee_2Eerror } A_{27t} A_{27w}) \text{ (c_2Ebool_2Ethe_value} \\
& \quad (\text{ty_2Epair_2Eprod } A_{27t} A_{27w})) \text{ (ap (ap c_2Ereal_2Ereal_sub} \\
& \quad (\text{ap (c_2Ebinary_ieee_2Efloat_to_real } A_{27t} A_{27w}) V0a)) \text{ (} \\
& \quad \text{ap (c_2Ebinary_ieee_2Efloat_to_real } A_{27t} A_{27w}) V1b))))))))) \\
& \hspace{15em} (80)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (82)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (83)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (84)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (85)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{87}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{89}$$

Theorem 1

$$\begin{aligned}
& \forall A_27t. nonempty A_27t \Rightarrow \forall A_27w. nonempty A_27w \Rightarrow (\\
& \forall V0a \in (ty_2Ebinary_ieee_2Efloat A_27t A_27w). (\forall V1b \in \\
& (ty_2Ebinary_ieee_2Efloat A_27t A_27w). (((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& A_27t A_27w) V0a)) \wedge ((p (ap (c_2Ebinary_ieee_2Efloat_is_finite \\
& A_27t A_27w) V1b)) \wedge (p (ap (ap c_2Erealx_2Ereal_lt (ap c_2Ereal_2Eabs \\
& (ap (ap c_2Ereal_2Ereal_sub (ap (c_2Ebinary_ieee_2Efloat_to_real \\
& A_27t A_27w) V0a)) (ap (c_2Ebinary_ieee_2Efloat_to_real A_27t \\
& A_27w) V1b)))) (ap (c_2Ebinary_ieee_2Ethreshold A_27t A_27w) \\
& (c_2Ebool_2Ethe_value (ty_2Epair_2Eprod A_27t A_27w)))))) \Rightarrow \\
& (p (ap (c_2Ebinary_ieee_2Efloat_is_finite A_27t A_27w) (ap \\
& (c_2Epair_2ESND ty_2Ebinary_ieee_2Eflags (ty_2Ebinary_ieee_2Efloat \\
& A_27t A_27w)) (ap (ap (ap (c_2Ebinary_ieee_2Efloat_sub A_27t \\
& A_27w) c_2Ebinary_ieee_2EroundTiesToEven) V0a) V1b))))))
\end{aligned}$$